Published by Institute of Physics Publishing for SISSA
Received: *May 31, 2005*
Accepted: *July 5, 2005*
Published: *August 2, 2005*

Large-volume flux compactifications: moduli spectrum and D3/D7 soft supersymmetry breaking

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Abstract: We present an explicit calculation of the spectrum of a general class of string models, corresponding to Calabi-Yau flux compactifications with $h_{1,2} > h_{1,1} > 1$ with leading perturbative and non-perturbative corrections, in which all geometric moduli are stabilised as in [14]. The volume is exponentially large, leading to a range of string scales from the Planck mass to the TeV scale, realising for the first time the large extra dimensions scenario in string theory. We provide a general analysis of the relevance of perturbative and non-perturbative effects and the regime of validity of the effective field theory. We compute the spectrum in the moduli sector finding a hierarchy of masses depending on inverse powers of the volume. We also compute soft supersymmetry breaking terms for particles living on D3 and D7 branes. We find a hierarchy of soft terms corresponding to 'volume dominated' F-term supersymmetry breaking. F-terms for Kähler moduli dominate both those for dilaton and complex structure moduli and D-terms or other de Sitter lifting terms. This is the first class of string models in which soft supersymmetry breaking terms are computed after fixing all geometric moduli. We outline several possible applications of our results, both for cosmology and phenomenology and point out the differences with the less generic KKLT vacua.

Keywords: Superstrings and Heterotic Strings, Superstring Vacua.

JHEP08(2005)007

Contents

JHEP08(2005)007

1. Introduction

We are entering a new phase in string phenomenology. There now exist well controlled mechanisms to stabilise the moduli fields and break supersymmetry, the two outstanding problems for low-energy string models [1 − 3]. Furthermore quasi-realistic models have been constructed that fit precisely within this moduli-fixing scenario. For the first time we can do proper phenomenology, starting with the computation of the spectrum of low-energy particles after having found a well-defined vacuum solution with all moduli stabilised. This programme has not been fully attempted so far. The main obstacle has been the lack of a concrete case in which the spectrum and soft supersymmetry breaking terms can actually be computed after stabilising all moduli.

Having a concrete model with all moduli fixed allows, first of all, the computation of all relevant scales from first principles: the string scale, Kaluza-Klein masses, gravitino mass, as well as masses of the different particles in the moduli sector. Furthermore, it allows the computation of the magnitude of soft supersymmetry breaking terms in the observable sector of the theory.

There have been several important attempts at computing soft supersymmetry breaking terms from flux compactifications in type-IIB string theory [4 − 7]. However, not all geometric moduli are fixed by the fluxes. The standard procedure to fix the remaining moduli, namely the KKLT scenario which involves the introduction of non-perturbative effects from D7 branes or D3 instantons, actually restores supersymmetry in an AdS minimum. Then in this scenario the breaking of supersymmetry is entirely due to the mechanism that lifts the minimum to de Sitter space, thus erasing the effects of fluxes in the calculation of the spectrum of moduli and matter fields. Recently there have been attempts to address this problem [8 − 11]. The main obstacle here is the lack of simple concrete realisations of the KKLT scenario in which soft breaking terms can be computed. In particular, in explicit models, what was a minimum in the complex structure and dilaton directions may become a saddle point in the potential after including the Kähler moduli [8, 9, 12] .

Nevertheless, this is only a manifestation of the model dependence of the KKLT scenario. There should be many models with varying numbers of complex structure moduli in which a concrete KKLT minimum can be found — for the state of the art see [13]. However, explicit control of the effective potential dependence on all the moduli tends to be lost once there are more than a few complex structure moduli. The lack of concrete calculations in this case is only a human limitation rather than a problem with the KKLT minimum.

Fortunately a large new class of minima of the full scalar potential, differing from the KKLT solutions, has been recently uncovered [14] (also see [15]). The main ingredient here is the realisation that after including both perturbative and non-perturbative corrections

JHEP08(2005)007

to the scalar potential, a very general minimum emerges which is non-tachyonic in all directions in geometric moduli space. General arguments show that this occurs for all Calabi-Yau compactifications with $h^{1,2} > h^{1,1} > 1$. A simple example with two Kähler moduli, the $\mathbb{P}^4_{[1,1,1,6,9]}$ model, was studied in detail and the minimum found explicitly. An important property of this minimum is that it is at exponentially large volume. This creates a naturally small expansion parameter, namely the inverse volume, which allows good control of otherwise intractable calculations. Having such a large volume gives rise to a stringy realisation of the large extra dimensions scenario [16]. It is then natural to study in detail this class of models and in particular the explicit two Kähler moduli case.

There are several important properties of these new vacua. They are generic and the corresponding field theory is valid for a much broader range of parameters than the KKLT minima. Furthermore, they lack the many tachyonic directions that often appear in the KKLT solutions and are thus more robust than the corresponding KKLT vacua. They are also physically very different. Besides the much larger volume, the AdS minimum is non-supersymmetric and therefore the sources of supersymmetry breaking are many: fluxes, non-perturbative effects and dS lifting mechanisms. In KKLT the AdS minimum is supersymmetric and the sole source of supersymmetry breaking is the dS lifting mechanism.

The purpose of this article is to start a complete phenomenological analysis of these models. We first compute the spectrum in the moduli sector as a function of the model parameters, in particular the volume. We obtain the volume dependence of the various mass scales present, and find a natural hierarchy between the string scale, Kaluza-Klein masses, gravitino mass, complex structure-dilaton masses and Kähler moduli masses. The mass scales depend on different inverse powers of the volume except for the axionic partner of the 'volume' modulus, whose mass is an inverse exponential of the volume and thus extremely small. We find that exponentially large volumes are generic and for $\mathcal{O}(1)$ values of the parameters (fluxes, terms in the non-perturbative superpotential, etc.) a wide range of string scales can be explicitly obtained, from the Planck mass to the electroweak and beyond. We also perform the corresponding analysis for the soft breaking terms for matter on D3/D7 branes and find a hierarchy of soft terms dominated by the F-term of the volume modulus.

The article is organised as follows. In section 2 we give a brief overview of flux compactifications of type-IIB string theory. In section 3 we present a general discussion of the limitations of the standard KKLT effective potential, caused by the neglect of the perturbative corrections to the Kähler potential. This restricts the volume to be not too large and the range of values of the flux superpotential to be extremely small. We point out that if the leading α'^3 corrections are included, the potential can be trusted for a far broader range of values. The robustness of the known α'^3 correction is critically investigated and we conclude that possible extra bulk corrections are further suppressed by negative powers of the volume. Section 4 reviews the class of models we consider and provides a detailed calculation of the magnitude of the relevant scales (string scale, Kaluza-Klein scale, gravitino mass, moduli and modulino masses). We explicitly compute the spectrum for the two Kähler moduli Calabi-Yau $\mathbb{P}^4_{[1,1,1,6,9]}$ mentioned above. We also present results for the general case, illustrating our conclusions with a three Kähler moduli example (the $\mathcal{F}_{11}$ model of [17]).

Section 5 concentrates on the soft supersymmetry breaking terms for both D3 and D7 matter fields. We find a hierarchy of supersymmetry breaking in which the F-term corresponding to the volume modulus dominates. The effects of D-terms or IASD fluxes for the de Sitter lift are also discussed. We conclude the article with a general comparison with the KKLT scenario, pointing out the similarities and differences. Finally we present our conclusions and general outlook.

2. Flux compactifications of IIB string theory

We shall first briefly review flux compactifications of type-IIB string theory mostly to establish conventions and to collect the basic equations we will be using and modifying later.

The ten-dimensional bosonic massless fields consist of the metric g_{MN}, scalars ϕ, C_0, RR antisymmetric tensors C_2, C_4, the latter with self-dual field strength, and the NS antisymmetric tensor B_2. To obtain four-dimensional $\mathcal{N} = 1$ models, we compactify on Calabi-Yau orientifolds. Fluxes for the RR three-form $F_3 = dC_2$ and NS three-form $H_3 = dB_2$ can be turned on, and must satisfy the standard quantisation conditions:

$$\frac{1}{(2\pi)^2\alpha'}\int_{\Sigma_a} F_3 = n_a \in \mathbb{Z}\,, \qquad \frac{1}{(2\pi)^2\alpha'}\int_{\Sigma_b} H_3 = m_b \in \mathbb{Z}\,, \tag{2.1}$$

where $\Sigma_{a,b}$ represent three-cycles of the Calabi-Yau manifold. The metric is then a warped product of flat 4-dimensional spacetime and a conformally Calabi-Yau orientifold, with the five-form field strength also dependent on the warp factor.

This compactification generically has $O3/O7$ orientifold planes, D3/D7 branes and fluxes. These all contribute to the C_4 tadpole that must be cancelled. This condition reads:

$$N_{D3} - N_{\bar{D}3} + \frac{1}{(2\pi)^4\alpha'^2}\int H_3 \wedge F_3 = \frac{\chi(X)}{24}\,. \tag{2.2}$$

Here $\chi(X)$ collects the contribution to D3 brane charge from orientifold planes and D7 branes. In the F-theory interpretation, $\chi(X)$ is the Euler number of the corresponding four-fold. Gauge theories live on the world-volume of both D3 and D7 branes, and can give rise to either standard model or hidden sector matter.

The effective field theory corresponds to a standard $\mathcal{N} = 1$ supergravity theory, with the superpotential being of Gukov-Vafa-Witten type [18]:

$$W = \int_M G_3 \wedge \Omega\,, \tag{2.3}$$

where $G_3 = F_3 - iSH_3$, with S[1] being the dilaton-axion

$$iS \equiv \tau = C_0 + ie^{-\phi}\,, \tag{2.4}$$

[1]We shall denote the dilaton-axion and Kähler moduli by S and T respectively. These are related to the τ and ρ frequently used by $\tau = iS$ and $\rho = iT$.

JHEP08(2005)007

and Ω the holomorphic $(3,0)$ form of the Calabi-Yau. This superpotential depends both on the dilaton, as it appears explicitly in the definition of G_3, and the complex structure moduli U, as these moduli measure the size of the 3-cycles that appear in the quantisation condition (2.1). However, W does not depend on the Kähler moduli.

The Kähler potential $\mathcal{K}$ is a sum of terms depending on the different moduli $\mathcal{K} = \mathcal{K}_{T_i} + \mathcal{K}_{U_a} + \mathcal{K}_S$[2] and takes the standard form (neglecting warp factors):

$$\mathcal{K} = -2\log[\mathcal{V}] - \log\left[-i\int_M \Omega\wedge\bar{\Omega}\right] - \log\left(S+\bar{S}\right), \tag{2.5}$$

where $\mathcal{V}$ is the classical volume of the Calabi-Yau manifold M in units of $l_s = 2\pi\sqrt{\alpha'}$:

$$\mathcal{V} = \int_M J^3 = \frac{1}{6}\kappa_{ijk}t^i t^j t^k\,. \tag{2.6}$$

Here J represents the Kähler class, and t_i, $i = 1,\ldots,h_{1,1}$ are moduli measuring the size of two-cycles. The corresponding four-cycle moduli τ_i are defined by:

$$\tau_i = \partial_{t_i}\mathcal{V} = \frac{1}{2}\kappa_{ijk}t^j t^k\,. \tag{2.7}$$

The complexified Kähler moduli are:

$$T_i \equiv -i\rho_i \equiv \tau_i + ib_i\,, \tag{2.8}$$

with the axionic fields b_i coming from the RR four-form. The Kähler potential $\mathcal{K}_{T_i}$ is of no-scale type, with $G^{i\bar{\jmath}}\mathcal{K}_i\mathcal{K}_{\bar{\jmath}} = 3$. Here we denote the Kähler metric by $G_{i\bar{\jmath}} \equiv \mathcal{K}_{i\bar{\jmath}} = \partial^2\mathcal{K}/\partial_i\phi\partial_{\bar{\jmath}}\phi$ and $G^{i\bar{\jmath}} = G^{-1}_{i\bar{\jmath}}$. Using this and the superpotential's indepedence of the Kähler moduli, it follows that the standard $\mathcal{N}=1$ supergravity scalar potential:

$$V = e^{\mathcal{K}}\left[G^{i\bar{\jmath}}D_iW\bar{D}_{\bar{\jmath}}\bar{W} - 3|W|^2\right], \tag{2.9}$$

with i,j running over all moduli, becomes

$$V_{no-scale} = e^{\mathcal{K}}G^{a\bar{b}}D_aW\bar{D}_b\bar{W}\,, \tag{2.10}$$

where a and b run over dilaton and complex-structure moduli only. As $V_{no-scale}$ is positive definite, we can locate the complex structure moduli at a minimum of the potential by solving

$$D_aW \equiv \partial_aW + (\partial_aK)W = 0\,. \tag{2.11}$$

This can be done for generic choices of the fluxes and we denote the value of W following this step as W_0.

This procedure fixes the complex structure moduli and the dilaton but leave the Kähler moduli undetermined. The dilaton and complex structure moduli are then integrated out to focus on an effective theory for the Kähler moduli. To also stabilise these fields non-perturbative effects have to be included. These effects mostly arise from the gauge theories

[2]We will usually denote the complex structure part of the Kahler potential by $\mathcal{K}_{cs} \equiv \mathcal{K}_{U_a}$.

JHEP08(2005)007

living on the D7 branes. For gauge fields on a D7 brane wrapping a four-cycle of size τ_i, the gauge kinetic function is

$$f_i = \frac{T_i}{2\pi}\,. \tag{2.12}$$

Similarly the gauge kinetic function for D3 brane gauge fields takes the universal form

$$f = \frac{S}{2\pi}\,. \tag{2.13}$$

Either euclidean D3 brane instantons [19] or D7 brane gaugino condensation [20] from an anomaly free gauge theory will naturally give rise to a non-perturbative superpotential.

Thus the full non-perturbative superpotential is expected to take the form

$$W = W_0 + \sum_i A_i\, e^{-a_i T_i}\,, \tag{2.14}$$

where A_i, a_i are model dependent constants. Substituting (2.14) into the scalar potential (2.9) generates a nontrivial minimum for the Kähler moduli, corresponding to the solution of the equations

$$D_i W \equiv \frac{\partial W}{\partial T_i} + W \frac{\partial \mathcal{K}}{\partial T_i} = 0\,. \tag{2.15}$$

As then $D_i W = 0$ for all moduli, this corresponds to a supersymmetric solution. The presence of the $-3|W|^2$ term in the scalar potential ensures that this minimum is clearly anti de Sitter.

There are several extant proposals to lift the minimum to a de Sitter vacuum. These all essentially add a positive definite source to the scalar potentials. The original proposal involved anti D3 brane tensions [3], but one could also use magnetic field fluxes on D7 branes which correspond to D-terms [21], or IASD fluxes generating F-terms [22]. The values of W_0, as well as the lifting term, have to be carefully chosen to give a minimum with vanishing cosmological constant.

The fixed complex structure moduli depend on the integers that define the fluxes. There are very many discrete flux choices, rendering this amenable to a statistical treatment. This has been carried out in several articles [12, 23, 24] (see also [25, 26]). The main results relevant for us are that:

(i) There are an exponentially large number of solutions.

(ii) There are regions of moduli space that act as attractors where many solutions concentrate, namely the regions close to conifold singularities.

(iii) The effective superpotential or, more properly, $e^{\mathcal{K}_{cs}}|W_0|^2$ is uniformly distributed. Thus every value is possible, up to a maximum determined by the tadpole cancellation condition ($W_{0,max} \sim 100$), but larger values of W_0 are more common than small values.

(iv) As $T \sim \ln(W_0)$, the number of solutions drops exponentially with the internal volume.

(v) Finally, solutions are distributed preferentially at strong coupling. To be more precise, $N(\text{solutions } |g_s < \epsilon) \sim \epsilon$.

Let us note that some of these statistical results can change substantially after introducing effects to fix the Kähler moduli.

3. Perturbative effects in $\mathcal{N} = 1$ supergravity

The model described in [14] and further studied below involves the α' corrections as a crucial ingredient. While these are always present and must be accounted for, this invites suspicion. It is often felt that a totalitarian principle applies: if some corrections are important, all are, and thus it is preferable to seek vacua for which all α' corrections can be neglected. We will address below the question of when this is possible, and show that in type-IIB flux compactifications the answer is 'almost never': the leading contribution to the scalar potential nearly always comes from perturbative α' effects. However, it is helpful to regard the type-IIB case as an example of more general behaviour.

3.1 General analysis

Let us start with an $\mathcal{N} = 1$ supergravity theory with tree-level Kähler potential $\mathcal{K}$ and tree-level superpotential W. In general $\mathcal{K}$ receives corrections at every order in perturbation theory, $\mathcal{K}_p$, and non-perturbative corrections $\mathcal{K}_{np}$, whereas W is not renormalised in perturbation theory and only receives non-perturbative corrections W_{np}. Therefore we can write[3]

$$\mathcal{K} = \mathcal{K}_0 + \mathcal{K}_p + \mathcal{K}_{np} \approx \mathcal{K}_0 + J\,, \tag{3.1}$$

$$W = W_0 + W_{np} \approx W_0 + \Omega\,, \tag{3.2}$$

where J represents the leading (perturbative) correction to $\mathcal{K}$ and Ω the leading non-perturbative correction to W in a coupling expansion. We ask when it is safe to neglect the corrections J or Ω.

The F-term scalar potential is

$$V = e^{\mathcal{K}} \left[D_i W \, D_{\bar{k}} \bar{W} \, (\mathcal{K})^{-1}_{i\bar{k}} - 3|W|^2 \right] . \tag{3.3}$$

This can be expanded in powers of J and Ω as follows:

$$V = V_0 + V_J + V_\Omega + \cdots \,, \tag{3.4}$$

where

$$V_0 \sim W_0^2\,, \qquad V_J \sim J W_0^2\,, \qquad V_\Omega \sim \Omega^2 + W_0 \Omega\,,$$

and the ellipses refer to higher-order terms combining J and Ω. The exact expressions for these quantities may be explicitly computed but are not relevant for most of our argument below.

[3]The quantities J and Ω introduced here should not be confused with the Calabi-Yau forms.

Normally, the structure of V is essentially determined by V_0, with the other terms providing small corrections in a weak coupling expansion. However if the tree-level potential has a flat direction along which V_0 is constant, then the structure of the potential, and in particular its critical points, are determined by the corrections. This behaviour is in fact common, with prime examples being the no-scale potentials generically appearing in string theory. These correspond to a Kähler potential satisfying $G^{-1}_{i\bar{k}}\mathcal{K}_i\mathcal{K}_{\bar{k}} = 3$ and a constant superpotential W_0. In this case $V_0 = 0$ and it is the corrections that determine the structure of the potential. Although both V_J and V_Ω will play a role, typically we expect V_J to dominate over V_Ω, as the former is perturbative in the coupling and the latter non-perturbative. However, since Ω is the only correction to W_0 it cannot be totally neglected.

In some, very special, cases it is safe to neglect the $\mathcal{O}(J)$ corrections. First, if $W_0 = 0$ then automatically $V_J = 0$ and the leading correction comes from V_Ω. For example, this occurs in the standard heterotic string racetrack scenario. Similarly, if $W_0 \ll 1$ in suitable units, then Ω may be of similar magnitude to W_0. In this case, we have

$$\Omega \sim W_0 \Rightarrow V_\Omega \sim \Omega^2 \qquad \text{and} \qquad V_J \sim J\Omega^2\,,$$

and therefore

$$\frac{V_J}{V_\Omega} \ll 1\,. \tag{3.5}$$

This is the relevant behaviour for the KKLT scenario. Finally, once $W > \Omega/J$ (which is $\ll 1$ as $\Omega \ll J$), the perturbative effects dominate and must be included.

It is worth noting that the limit $W \sim \Omega$, in which the tree level superpotential is comparable to its non-perturbative corrections, is very unnatural. There is furthermore no need to restrict to this limit if we have information on the perturbative corrections to $\mathcal{K}$, which is true in both type-IIB and heterotic cases.[4]

3.2 Application to type-IIB flux compactifications

Let us illustrate these issues in the concrete setting of type-IIB flux compactifications. As discussed above, the Kähler and superpotentials take the forms,

$$\begin{aligned} W &= \frac{1}{l_s^2}\int G_3 \wedge \Omega + \sum A_i e^{-a_i T_i}\,, \\ \mathcal{K} &= -2\ln(\mathcal{V}_E) - \ln\left(-i\int \Omega\wedge\bar{\Omega}\right) - \ln(S+S^*)\,. \end{aligned} \tag{3.6}$$

There is also a normalisation factor in front of W that is not important here but will be considered in section 4. The volume $\mathcal{V}_E$ and moduli T_i are measured in Einstein frame

[4]Note that the original proposal for gaugino condensaton in the heterotic string [20] included a constant term in the superpotential from the antisymmetric tensor H_{mnp} as well as the nonperturbative, gaugino condensation superpotential for the dilaton. This was abandoned because the constant was found to be quantised in string units and could not be of order the nonperturbative correction. However, since the leading perturbative corrections to K were found soon after, this avenue could have been revived in the early 90's.

JHEP08(2005)007

$(g_{\mu\nu,E} = e^{-\frac{\phi}{2}} g_{\mu\nu,s})$. The non-perturbative superpotential is generated by either D3-brane instantons ($a_i = 2\pi$) or gaugino condensation ($a_i = \frac{2\pi}{N}$). If the dilaton and complex structure moduli have been fixed by the fluxes, these potentials reduce to

$$\begin{aligned} W &= W_0 + \sum A_i e^{-a_i T_i}\,, \\ \mathcal{K} &= \mathcal{K}_{cs} - 2\ln(\mathcal{V}_E)\,. \end{aligned} \tag{3.7}$$

In the language of section 3.1, the scalar potential derived from (3.7) includes V_Ω but not V_J. We now show that for almost all flux choices and moduli values, the use of (3.7) without α' corrections is inconsistent. Although the argument extends easily to any number of Kähler moduli, for illustration we consider one Kähler modulus and the geometry appropriate to the quintic. (3.7) then reduces to

$$\begin{aligned} W &= W_0 + A e^{-aT}\,, \\ \mathcal{K} &= \mathcal{K}_{cs} - 3\ln(T+\bar{T})\,. \end{aligned} \tag{3.8}$$

For the quintic $\mathcal{V}_E = \frac{5}{6}t^3 = \frac{1}{6\sqrt{5}}(T+\bar{T})^{\frac{3}{2}} = \frac{\sqrt{2}}{3\sqrt{5}}\sigma^{\frac{3}{2}}$, where $T = \sigma + ib$. The leading α' correction to the Kähler potential is [28]

$$\mathcal{K}_{\alpha'} = \mathcal{K}_{cs} - 2\ln\left(\mathcal{V}_E + \frac{\xi}{2g_s^{\frac{3}{2}}}\right), \tag{3.9}$$

where $\xi = \frac{-\chi(M)\zeta(3)}{2(2\pi)^3} = 0.48$. The factor of $g_s^{-\frac{3}{2}}$ arises from our working in Einstein frame; it would be absent in string frame, in which $\mathcal{V}_s = \mathcal{V}_E g_s^{\frac{3}{2}}$. The resulting scalar potential is

$$V = e^{\mathcal{K}}\left(\overbrace{\frac{4\sigma^2(Aa)^2 e^{-2a\sigma}}{3} - 4\sigma W_0 (Aa)e^{-a\sigma}}^{V_\Omega} + \overbrace{\frac{9\sqrt{5}\xi W_0^2}{4\sqrt{2}g_s^{\frac{3}{2}}\sigma^{\frac{3}{2}}}}^{V_J}\right). \tag{3.10}$$

The α' correction V_J dominates at both small and large volume. Although perhaps counter-intuitive, this is to be expected - the α' correction is perturbative in volume whereas the competing terms are non-perturbative.

We may quantify when neglecting α' corrections is permissible. The allowed range of σ is

$$\mathrm{Max}(\mathcal{O}\left(\frac{1}{g_s}\right), \sigma_{\min}) < \sigma < \sigma_{\max}\,, \tag{3.11}$$

where $\sigma_{\min}$ and $\sigma_{\max}$ are taken to be the solutions of $|V_J(\sigma)| = |V_\Omega(\sigma)|$, that is

$$\frac{4\sigma^2|A^2\,a^2|e^{-2a\sigma}}{3} - 4\sigma|W_0 A\,a|e^{-a\sigma} = \frac{9\sqrt{5}\xi|W_0|^2}{4\sqrt{2}g_s^{\frac{3}{2}}\sigma^{\frac{3}{2}}}\,. \tag{3.12}$$

The $\mathcal{O}(1/g_s)$ bound on σ comes from requiring $\mathcal{V}_s > 1$ in order to control the α' expansion. In general this regime is rather limited. For very moderate values of W_0, equation (3.12) has no solutions and there is *no* region of moduli space in which it is permissible to neglect α' corrections to (3.8).

For concreteness let us consider superpotentials generated by D3-brane instantons and let us take[5] $A = 1$ and $g_s = 1/10$. Then we require $W_0 < \mathcal{O}(10^{-75})$ to have any solutions at

[5]Properly this should be $e^{\mathcal{K}_{cs}/2}A$ to be Kähler covariant, but we shall often drop the $e^{\mathcal{K}_{cs}/2}$.

all of (3.12) with $\mathcal{V}_s > 1$. This can be improved somewhat by using gaugino condensation with very large gauge groups. Thus using $W_0 = 10^{-5}$, $g_s = 1/10$ and $N = 50$, $\sigma_{\max} \sim 150$, which corresponds to $\mathcal{V}_s \sim 12$. However, as generic values of W_0 are $\mathcal{O}(\sqrt{\chi/24}) \sim \mathcal{O}(10)$ and W_0^2 is uniformly distributed [17], $W_0 = 10^{-5}$ represents a tuning of one part in 10^{12}, and even then the range of validity is rather limited.

Our conclusion is then that for generic values of W_0 there is no regime in which the perturbative corrections to the Kähler potential can be neglected; the inclusion solely of non-perturbative corrections is inconsistent. Furthermore, even for the small values of W_0 for which there does exist a regime in which non-perturbative terms are the leading corrections, this is still only true for a small range of moduli values and in particular never holds at large volume.[6]

3.3 Systematics of the α' expansion

The above arguments show that in KKLT-type flux compactifications, under almost all circumstances the neglect of α' corrections is inconsistent. There are two conclusions to be drawn from this. First, to investigate the structure of the potential at large volumes we must include the α' corrections. It is thus in principle impossible to ask whether type-IIB flux compactifications can realise the large extra dimensions scenario without including these corrections. Secondly, the inclusion of the α' corrections is necessary for the study of the type-IIB flux landscape and its statistics. Models with all moduli stabilised are a precondition for meaningful discussions of the landscape and its properties, and the α' corrections are required to discuss moduli stabilisation across almost the entirety of Kähler moduli space. While there exist examples for which the Kähler moduli can be stabilised purely by non-perturbative effects [13], such models are unlikely to be representative, not least because all such minima are supersymmetric.

Fortunately, the inclusion of α' corrections does not require full knowledge of the string theory. At small volumes, the α' corrections do appear democratically, and it would be difficult to extract reliable results. However, the α' expansion is at heart an expansion in inverse volume. At very large volumes, the expansion parameter for α' effects is $\frac{1}{\mathcal{V}}$, and a systematic inclusion of such effects will give a controlled expansion.

Let us treat this more explicitly by studying the volume scaling of the terms arising from dimensional reduction of the α'-corrected type-IIB supergravity action. Of course this action is not known fully, but our arguments will only depend on the general form of the terms rather than the specific details of the tensor structure. The supergravity action consists of bulk and localised terms and is

$$S_{IIB} = S_{b,0} + \alpha'^3 S_{b,3} + \alpha'^4 S_{b,4} + \alpha'^5 S_{b,5} + \cdots + S_{cs} + S_{l,0} + \alpha'^2 S_{l,2} + \cdots . \tag{3.13}$$

The localised sources present are D3/D7 or O3/O7 planes. In string frame, we have [81]

$$S_{b,0} = \frac{1}{(2\pi)^7 \alpha'^4} \int d^{10}x \sqrt{-g} \left\{ e^{-2\phi} [\mathcal{R} + 4(\nabla\phi)^2] - \frac{F_1^2}{2} - \frac{1}{2 \cdot 3!} G_3 \cdot \bar{G}_3 - \frac{\tilde{F}_5^2}{4 \cdot 5!} \right\},$$

[6] We do not consider the special case $\chi(M) = 0$, when the α'^3 corrections considered above vanish. We do not know the status of higher α' corrections in such models.

$$
\begin{aligned}
S_{cs} &= \frac{1}{4i(2\pi)^7\alpha'^4}\int e^{\phi} C_4 \wedge G_3 \wedge \bar{G}_3 \,, \\
S_{l,0} &= \sum_{\text{sources}} \left(-\int d^{p+1}\xi \, T_p \, e^{-\phi}\sqrt{-g} + \mu_p \int C_{p+1} \right) .
\end{aligned}
\tag{3.14}
$$

We may avoid the need to include D3-branes by taking the fluxes to saturate the C_4 tadpole. We shall work throughout in the F-theory orientifold limit, in which the dilaton is constant: $\tau = \tau(y) = \tau_0$.

For flux compactifications with ISD 3-form fluxes, the metric and fluxes take the form [2]

$$
\begin{aligned}
ds^2_{10} &= e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2A(y)}\tilde{g}_{mn}dy^m dy^n \,, \\
\tilde{F}_5 &= (1+*)\left[d\alpha \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \right] , \\
F_3, H_3 &\in H^3(M,\mathbb{Z}) \,,
\end{aligned}
\tag{3.15}
$$

where $\alpha = e^{4A}$ parametrises both the magnitude of the warping and the size of the 5-form flux. Here $\tilde{g}$ is a Calabi-Yau metric; the flux back-reacts to render the compact space only conformally Calabi-Yau. The warp-factor transforms non-trivially under internal rescalings, with warping effects suppressed at large volume. Specifically, under $\mathcal{V} \to \lambda^6 \mathcal{V}$, where $\lambda \gg 1$, $\alpha = 1 + \mathcal{O}(\frac{1}{\lambda^4}) + \cdots$.

We first consider contributions tree level in α'. The leading contribution to the 4-d scalar potential arises from the flux term $\frac{1}{(2\pi)^7\alpha'^4}\int G_3 \cdot \bar{G}_3$, which gives

$$
V_{\text{flux}} \sim \mathcal{K}^{a\bar{b}}_{cs}\frac{D_a W D_{\bar{b}} W}{\mathcal{V}^2} \,,
\tag{3.16}
$$

where the sum is over dilaton and complex structure moduli. This term is positive semi-definite and vanishes at its minimum. The volume scaling is understood as follows:

$$
V_{\text{flux}} \sim \overbrace{\mathcal{V}^{-2}}^{\text{conversion to Einstein frame}} \times \overbrace{\mathcal{V}}^{\text{internal integral}} \times \overbrace{\mathcal{V}^{-1}}^{G_3 \cdot \bar{G}_3} \sim \mathcal{V}^{-2} \,.
\tag{3.17}
$$

In the absence of warping, (3.16) is the only $\mathcal{O}(\alpha'^0)$ contribution to the potential energy, as $\tilde{F}_5 = \mathcal{R} = 0$ and dilaton gradients vanish. However, there is also a warping contribution. At large volume, $\alpha \sim 1 + \frac{1}{\mathcal{V}^{\frac{2}{3}}}$ and thus $V_{F_5} = \int d^6x\sqrt{g}F_5^2 \sim \int d^6x\sqrt{g}(\mathrm{d}\alpha)^2$ contributes

$$
V_{F_5} \sim \overbrace{\mathcal{V}^{-2}}^{\text{conversion to Einstein frame}} \times \overbrace{\mathcal{V}}^{\text{internal integral}} \times \overbrace{\mathcal{V}^{-\frac{5}{3}}}^{F_5^2} \sim \mathcal{V}^{-8/3} \,.
\tag{3.18}
$$

As now $\mathcal{R} \neq 0$, the Einstein-Hilbert term $\int_{Y_6}\sqrt{-g}\mathcal{R}$ is also important and in fact contributes identically with V_{F_5}. These terms may be related to the tree-level flux term and serve as an additional prefactor. The net result is [56]

$$
V_{0,\text{unwarped}} = \frac{1}{2\kappa^2_{10}\,\mathrm{Im}\,\tau}\int_M G_3^+ \wedge *_6 \bar{G}_3^+ \to V_{0,\text{warped}} = \frac{1}{2\kappa^2_{10}\mathrm{Im}\,\tau}\int_M e^{4A} G_3^+ \wedge *_6 \bar{G}_3^+ \,.
$$

It is important that this potential remains no-scale, with no potential generated for the Kähler moduli.

The bulk effective action receives higher-derivative corrections starting at $\mathcal{O}(\alpha'^3)$, which is also the order at which string loop corrections first appear; the tree level action is already $\mathrm{SL}(2,\mathbb{Z})$ invariant and receives no g_s corrections. The discussion of loop corrections to S_b is thus subsumed into the discussion of higher-derivative corrections.

The bosonic fields are the metric, dilaton-axion, 3-form field strength G_3 and self-dual five form field strength F_5. While its precise form is unknown, $S_{b,3}$ is expected to include all combinations of these consistent with the required dimensionality. At $\mathcal{O}(\alpha'^3)$, the bosonic action takes the schematic form

$$S_{b,3} \sim \frac{\alpha'^3}{\alpha'^4} \int d^{10}x\sqrt{-g}\Big[\Big(\mathcal{R}^4 + \mathcal{R}^3\left(G_3G_3 + G_3\cdot\bar{G}_3 + G_3\bar{G}_3 + F_5^2 + \partial\tau\cdot\partial\tau + \nabla^2\tau\right) + \\ + \mathcal{R}^2((DG_3)^2 + (DF_5)^2 + G^4 + \cdots) + \\ + \mathcal{R}(G_3^6 + \cdots) + (G_3^8 + \cdots)\Big)\Big].$$

Terms linear in the fluxes (e.g. $\mathcal{R}^3 DG_3$) are forbidden as the action must be invariant under world-sheet parity. The tensor structure and modular behaviour of the majority of these terms is unknown. A notable exception is the $\mathcal{R}^4$ term, the coefficient of which is known exactly to be an Eisenstein series in the dilaton [27]. However, our interest is in the volume scaling, which can be extracted on merely dimensional grounds.

Let us first consider terms independent of warping. The $\mathcal{R}^3(G^2 + c.c.)$ and $\mathcal{R}^3 G\bar{G}$ terms are most easily understood. These are similar to the $\mathcal{O}(\alpha'^0)$ $G_3\bar{G}_3$ term but with three extra powers of curvature. Then

$$V_{\mathcal{R}^3G^2} \sim \overbrace{\mathcal{V}^{-2}}^{\text{Einstein frame}} \times \overbrace{\mathcal{V}}^{\text{internal integral}} \times \overbrace{\mathcal{V}^{-1}}^{\mathcal{R}^3} \times \overbrace{\mathcal{V}^{-1}}^{G_3^2} \sim \mathcal{V}^{-3}. \tag{3.19}$$

The same argument tells us that a similar scaling applies for $\mathcal{R}^2(DG_3)^2$ terms, whereas $\mathcal{R}^2G^4$ terms contribute as $\sim \frac{1}{V^{11/3}}$, $\mathcal{R}G^6$ terms as $\sim \frac{1}{V^{13/3}}$ and G^8 terms as $\sim \frac{1}{V^5}$. In the absence of warping, the $\mathcal{R}^4$ term does not contribute to the potential energy; integrated over a Calabi-Yau, it vanishes. This geometric result can be understood macroscopically; were this not to vanish, it would generate a potential for the volume even in flux-less $\mathcal{N} = 2$ IIB compactifications. However, it is known there that tree level moduli remain moduli to all orders in the α' and g_s expansions, and thus higher derivative terms must not be able to generate a potential for them. This term contributes indirectly by modifying the dilaton equations of motion at $\mathcal{O}(\alpha'^3)$; we will return to this later.

There are also higher-derivative terms dependent on the warp factor. Examples are

$$\frac{\alpha'^3}{\alpha'^4}\int\sqrt{g}\mathcal{R}^4\,, \qquad \frac{\alpha'^3}{\alpha'^4}\int\sqrt{g}\mathcal{R}^3F_5^2\,, \qquad \frac{\alpha'^3}{\alpha'^4}\int\sqrt{g}\mathcal{R}^2(DF_5)^2\,.$$

These are similar to the corresponding tree-level terms but with three extra powers of curvature. As $\mathcal{R}^3 \sim 1/\mathcal{V}$, the contribution of such terms is no larger than $\mathcal{O}(\frac{1}{\mathcal{V}^{11/3}})$.

There are also potential contributions from internal dilaton gradients. As we have worked in the orientifold limit of F-theory, we have thus far regarded such terms as vanishing. However, this is not quite true. In the presence of higher derivative corrections, a constant dilaton no longer solves the equations of motion. Instead, we have

$$\phi(y) = \phi_0 + \frac{\zeta(3)}{16} Q(y) \,. \tag{3.20}$$

An explicit expression for $Q(y)$ may be found in [28], but for our purposes it is sufficient to note that $Q \sim \mathcal{R}^3 \sim \frac{1}{\mathcal{V}}$. It is then easy to see that terms such as

$$\int (\partial\tau \cdot \partial\bar{\tau}) \mathcal{R}^2 G^2 \qquad \text{or} \qquad \int (\nabla^2 \phi)(DF_5)^2 \mathcal{R}$$

are suppressed compared to the terms considered above. Note that dilaton-curvature terms such as

$$\int (\nabla^2 \phi) \mathcal{R}^3$$

will not contribute to the potential energy; these exist in $\mathcal{N} = 2$ Calabi-Yau compactifications and so must vanish either directly or by cancellation. Similar comments apply for the fact that, even without warping, the internal space ceases to be Calabi-Yau at $\mathcal{O}(\alpha'^3)$.

There is one further effect associated with (3.20). As the dilaton is no longer constant, under dimensional reduction the four-dimensional Einstein-Hilbert term is renormalised. Rescaling this to canonical form introduces a term $\frac{V_{\text{tree}}}{\mathcal{V}}$ of $\mathcal{O}(\frac{1}{\mathcal{V}^3})$. However, as V_{tree} is no-scale this correction does not break the no-scale structure and in particular vanishes at the minimum.

String loop corrections first appear at $\mathcal{O}(\alpha'^3)$ and are thus subsumed into the above analysis. While it may be difficult to derive anything explicitly, we may conjecture their effect in the large volume limit. The corrections to the Kähler potential arise from dimensional reduction of the 10-dimensional $\mathcal{R}^4$ term. In Einstein frame, the string tree-level α'^3 corrected Kähler potential derived in [28] takes the form

$$\mathcal{K} = \mathcal{K}_{cs} - 2\ln\left(\mathcal{V} + \frac{\xi}{2e^{3\phi/2}}\right) . \tag{3.21}$$

Here $\xi = -\frac{\chi(M)\zeta(3)}{2(2\pi)^3}$. $\zeta(3)$ is distinctive as the tree-level coefficient of the ten-dimensional $\mathcal{R}^4$ term, the coefficient of which is known exactly to be

$$f^{(0,0)}_{\frac{3}{2}}(\tau,\bar{\tau}) = \sum_{(m,n)\neq(0,0)} \frac{e^{-\frac{3\phi}{2}}}{|m+n\tau|^3} \,. \tag{3.22}$$

This has the expansion

$$f^{(0,0)}_{\frac{3}{2}}(\tau,\bar{\tau}) = \frac{2\zeta(3)}{e^{\frac{3\phi}{2}}} + \frac{2\pi^2}{3} e^{\frac{\phi}{2}} + \text{instanton terms} \,. \tag{3.23}$$

Therefore, to incorporate $\mathcal{O}(\alpha'^3)$ string loop corrections to S_{IIB}, the natural conjecture is that we should modify (3.21) to

$$\mathcal{K} = \mathcal{K}_{cs} - 2\ln\left(\mathcal{V} - \frac{\chi(M)}{8(2\pi)^3} f^{(0,0)}_{\frac{3}{2}}(\tau,\bar{\tau})\right) . \tag{3.24}$$

Let us finally mention further higher derivative corrections at $\mathcal{O}(\alpha'^4)$ and above. At large volume these are all subleading, with any terms generated being subdominant to the $1/\mathcal{V}^3$ terms present at $\mathcal{O}(\alpha'^3)$. For example, an $\mathcal{O}(\alpha'^4)$ term $G^2\mathcal{R}^4$ would give a $\mathcal{V}^{-\frac{10}{3}}$ contribution to the potential. There are other terms that would naively give a $1/\mathcal{V}^3$ contribution, such as a possible $\mathcal{O}(\alpha'^6)$ term $\mathcal{R}^6$. However, on a Calabi-Yau such a term vanishes, either explicitly or through cancellation. This is for the reasons discussed above: terms present even in pure $\mathcal{N}=2$ Calabi-Yau compactification cannot generate potentials for the moduli.

Our conclusion is therefore that the leading α' corrections breaking the no-scale structure appear at $\mathcal{O}(\frac{1}{\mathcal{V}^3})$, coming from $\mathcal{R}^3G^2$ and $\mathcal{R}^2(DG)^2$ terms in ten dimensions. This is consistent with the result of [28], where the corresponding correction (3.21) to the Kähler potential was computed and the resulting scalar potential interpreted as descending from such ten-dimensional terms. Although the scaling argument used above cannot reproduce the coefficient of the correction, it makes it easier to see that other terms are subleading; in particular, the effects associated with non-vanishing F_5 (which were not considered in [28]) do not compete.

Let us now consider higher derivative corrections to localised sources. The D3-brane action is

$$S_{D3} = -T_3 \int d^4x\sqrt{-g}e^{-\phi} + \mu_3 \int C_4 \,. \tag{3.25}$$

As D3-branes are space-filling, any higher derivative corrections involve space-time curvature and so cannot give a potential energy. Furthermore, it is known that the D3-brane solution is unaltered by the α'^3 flux and curvature terms [29]. The α' corrections on D7-branes do give potential energy contributions; fortunately in that case such effects were already included in the analysis of [2]. We briefly review the relevant results. The leading α' correction to the D7-brane Chern-Simons action is (we do not turn on internal D7-brane fluxes)

$$S_{D7,\alpha'^2} = \frac{\mu_7}{96}(2\pi\alpha')^2 \int_{R^4\times\Sigma} C_4 \wedge \mathrm{Tr}(\mathcal{R}_2 \wedge \mathcal{R}_2)\,, \tag{3.26}$$

whereas the leading correction to the wrapped D7-brane tension arises from a term

$$\frac{-\mu_7}{96}(2\pi\alpha')^2 \int_{R^45\times\Sigma} \sqrt{-g}\mathrm{Tr}(\mathcal{R}_2 \wedge *\mathcal{R}_2)\,. \tag{3.27}$$

These contribute effective D3-brane charge and tension. In F-theory, the D3-brane charge from the wrapped D7-branes is

$$Q_{D3} = -\frac{\chi(X)}{24}\,, \tag{3.28}$$

where X is the Calabi-Yau fourfold. As the D7 branes are BPS, (3.28) also gives the resulting effective D3 tension.

A similar argument also shows that higher α' corrections to the D7-brane action need not be considered. As the branes are BPS, α' corrections to the tension are related to α' corrections to the charges. However, these involve spacetime curvature, and so the corresponding correction to the DBI action cannot give rise to a 4d potential energy.

JHEP08(2005)007

We have focused above on α' corrections from bulk and localised sources, and g_s corrections from the bulk. We should finally consider string loop corrections arising from the open string sector. Less is known about these effects and it would be very interesting to study them further (see [30]). However, there is a sense in which such effects could only strengthen our results. We have already shown that the use solely of non-perturbative effects to describe the moduli potential is generically inconsistent due to the competing bulk α'^3 effects; extra corrections, if they exist, would further enhance the importance of perturbative effects.

4. Spectrum in the moduli sector

4.1 A working model

We have shown above both that α' corrections must be included to study Kähler moduli stabilisation, and that at large volume we only need include the leading corrections of [28]. In [14] a general analysis of the resulting potential was carried out. It was shown that there exists a special decompactification direction in moduli space along which the potential vanishes from below, with an associated minimum at exponentially large volumes. This behaviour was exhibited explicitly in a particular model, flux compactificatons on the orientifold of $\mathbb{P}^4_{[1,1,1,6,9]}$.

The general argument for this behaviour will be reviewed and given a geometric interpretation in section 4.6; however, we shall first extend our study of the concrete example. In particular, we shall calculate the spectrum of scales and particle masses, as well as the soft supersymmetry breaking terms.

$\mathbb{P}^4_{[1,1,1,6,9]}$ has $h^{1,1} = 2$ and $h^{2,1} = 272$ and its Kähler sector has been studied in [17]. In terms of the Kähler moduli, which following the notation of [17] we denote by T_4 and T_5, the volume may be written as

$$\mathcal{V} = \frac{1}{9\sqrt{2}}\left(\tau_5^{\frac{3}{2}} - \tau_4^{\frac{3}{2}}\right), \tag{4.1}$$

where $T_4 = \tau_4 + ib_4$ and $T_5 = \tau_5 + ib_5$. We fix the dilaton and complex structure moduli using 3-form fluxes. The periods are known [31] and this can be done straightforwardly as in [32−34]. We then integrate these out and concentrate on the Kähler moduli. It was shown in [17] that the divisors $\mathcal{D}_i$ corresponding to τ_4 and τ_5 have $\chi(\mathcal{D}) = 1$ and thus non-perturbative superpotentials will be generated for these moduli [19].

The condition $\chi(D) = 1$ for D a vertical divisor in the absence of flux is a sufficient but not a necessary condition for divisors to contribute to a superpotential. In the presence of fluxes, instanton zero modes can be lifted and non-perturbative superpotentials generated under more general circumstances. The exact necessary conditions are not yet known, but the trend of recent results [35−40] is that the condition $\chi(D) = 1$ will be substantially relaxed. However, in the model we study the non-perturbative superpotentials exist even under the most conservative requirement.

JHEP08(2005)007

After integrating out dilaton and complex structure moduli, the resulting superpotential is

$$W = W_0 + A_4 e^{-\frac{a_4}{g_s}T_4} + A_5 e^{-\frac{a_5}{g_s}T_5}\,. \tag{4.2}$$

There is a numerical prefactor of $g_s^2 M_P^3$ in (4.2) which will be important for the discussion of moduli masses in the next section. W_0 is determined by the fluxes; while tunable in principle, we impose no requirement that it be small. The Kähler potential is

$$\mathcal{K} = \mathcal{K}_{cs} - 2\ln\left(\mathcal{V} + \frac{\xi}{2}\right), \tag{4.3}$$

where the volume $\mathcal{V}$ is given by (4.1). Here $g_s = \langle e^{\phi}\rangle$ and Einstein frame has been defined by $g_{\mu\nu,s} = e^{(\phi-\phi_0)/2} g_{\mu\nu,E}$ (note that this convention differs from our usage in section 3.2).

Given (4.2) and (4.3), the scalar potential (3.3) may be computed directly. In the limit where $\tau_5 \gg \tau_4 > 1$, this takes the form

$$V = \frac{\lambda\sqrt{\tau_4}(a_4 A_4)^2 e^{-2\frac{a_4\tau_4}{g_s}}}{\mathcal{V}} - \frac{\mu W_0 (a_4 A_4)\tau_4 e^{-\frac{a_4\tau_4}{g_s}}}{\mathcal{V}^2} + \frac{\nu\xi W_0^2}{\mathcal{V}^3}\,, \tag{4.4}$$

where λ, μ and ν are model-dependent numerical constants. The axion partner of τ_4 will adjust to render the middle term of (4.4) negative; this adjustment reduces the potential, and will always occur. At large volumes we would naively expect the potential to be uniformly positive, due to the perturbative α'^3 term, but if τ_4 is logarithimically small compared to τ_5 this need not be true. Consider the decompactification limit

$$\tau_5 \to \infty \qquad \text{and} \qquad a_4 A_4 e^{-\frac{a_4\tau_4}{g_s}} = \frac{W_0}{\mathcal{V}}\,.$$

In this limit the potential (4.4) becomes

$$V = \frac{W_0^2}{\mathcal{V}^3}\left(\lambda\sqrt{\ln(\mathcal{V})} - \mu\ln\mathcal{V} + \xi\nu\right) \tag{4.5}$$

and thus approaches zero from below. Negativity of the potential requires that $\ln\mathcal{V}$ be large, and so associated with this behaviour there exists a minimum at exponentially large volumes. This is illustrated in figure 1. It is possible to solve

$$\frac{\partial V}{\partial\tau_4} = \frac{\partial V}{\partial\tau_5} = 0$$

analytically, to obtain

$$\tau_4 \propto \xi^{\frac{2}{3}} \qquad \text{and} \qquad \langle\mathcal{V}\rangle \propto W_0 e^{\frac{a_4\tau_4}{g_s}}\,. \tag{4.6}$$

It is necessary that this remains a minimum of the full potential including the dilaton and complex structure moduli. These are flux-stabilised and enter the potential as $\frac{DW\cdot DW}{\mathcal{V}^2}$; at the minimum this term vanishes, but is otherwise $\mathcal{O}(\frac{1}{\mathcal{V}^2})$. However, the vacuum energy at the minimum is $-\mathcal{O}(\frac{1}{\mathcal{V}^3})$. This ensures that the minimum is a real minimum of the full potential: movement of the complex structure moduli away from their stabilised values contributes a term $+\mathcal{O}(\frac{1}{\mathcal{V}^2})$ to the potential, but the negative terms cannot compete with this, and so this can only increase the potential. Therefore the minimum remains a minimum of the full potential.[7]

[7]In generic Calabi-Yau orientifold compactifications, in addition to the dilaton, complex structure and

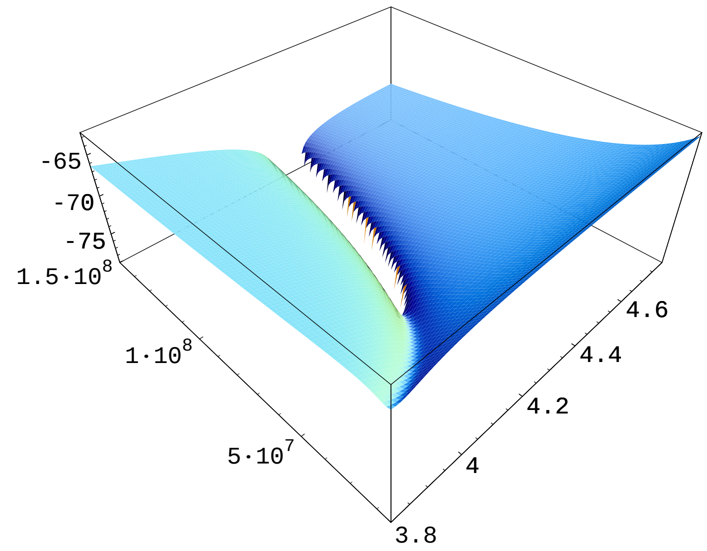

Figure 1: $\ln(V)$ for $\mathbb{P}^4_{[1,1,1,6,9]}$ in the large volume limit, as a function of the divisors τ_4 and τ_5. The void channel corresponds to the region where V becomes negative and $\ln(V)$ undefined. As $V \to 0$ at infinite volume, this immediately shows that a large-volume minimum must exist.

4.2 Scales and moduli masses

We set $\hbar = c = 1$ but will otherwise be pedantic on frames and factors of 2π and α'. Our basic length will be $l_s = 2\pi\sqrt{\alpha'}$ and our basic mass $m_s = \frac{1}{l_s}$. These represent the only dimensionful scales, and unless specified otherwise volumes are measured in units of l_s. We will furthermore require that at the minimum the 4-dimensional metric is the metric the string worldsheet couples to.

Stringy excitations then have

$$m_S^2 = \frac{n}{\alpha'} \Rightarrow m_S \sim 2\pi m_s \,. \tag{4.7}$$

To estimate Kaluza-Klein masses we first recall toroidal compactifications. A stringy ground state of Kaluza-Klein and winding integers n and w has mass

$$m_{KK}^2 = \frac{n^2}{R^2} + \frac{w^2 R^2}{\alpha'^2} \,, \tag{4.8}$$

where R is the dimensionful Kaluza-Klein radius. Strictly (4.8) only holds for toroidal compactifications, but it should suffice to estimate the relevant mass scale. If we write $R = R_s l_s$ and assume $R_s \gg 1$, we have

$$m_{KK} \sim \frac{m_s}{R_s} \qquad \text{and} \qquad m_W \sim (2\pi)^2 R_s m_s \,. \tag{4.9}$$

Kähler moduli, the resulting four dimensional low energy theory also has $h^{1,1}_-$ moduli coming from the reduction of the type IIB 2-forms B_2 and C_2. We will assume throughout that these are not present; in particular, this is true for the $\mathbb{P}^4_{[1,1,1,6,9]}$ model we discuss below.

It is conceivable that the geometry of the internal space is elongated such that the Kaluza-Klein radius R_s is uncorrelated with the overall volume, and we would then have KK masses of order $m_{KK}^4 \sim 1/\tau_i$ for the different cycles. However, in absence of evidence to the contrary we assume the simplest scenario in which $(2\pi R_s)^6 = V_s$. Then

$$m_{KK} \sim \frac{2\pi m_s}{\mathcal{V}_s^{\frac{1}{6}}} . \tag{4.10}$$

Here m_{KK} refers only to the lightest KK mode, as in our situation the overall volume is large but there are relatively small internal cycles. Therefore, while there may be many KK modes, there is a hierarchy with the others being naturally heavier than the scale of (4.10).

We next want to determine the masses of the complex structure and Kähler moduli. This requires an analytic expression for the potential in terms of canonically normalised fields. To obtain the potential, we will work in the framework of $\mathcal{N} = 1$ supergravity; the validity of this effective field theory approach is discussed in section 4.5. The dimensional reduction of the 10-dimensional action into this framework is carried out in more detail in appendix A; here we shall just state results.

An $\mathcal{N} = 1$ supergravity is completely specified by a Kähler potential, superpotential and gauge kinetic function. Neglecting the gauge sector to focus on moduli dynamics, the action is

$$S_{\mathcal{N}=1} = \int d^4x\sqrt{-G}\left[\frac{M_P^2}{2}\mathcal{R} - \mathcal{K}_{,i\bar{j}}D_\mu\phi^i D^\mu\bar{\phi}^j - V(\phi,\bar{\phi})\right], \tag{4.11}$$

where

$$V(\phi,\bar{\phi}) = e^{\mathcal{K}/M_P^2}\left(\mathcal{K}^{i\bar{j}}D_i\hat{W}D_{\bar{j}}\bar{\hat{W}} - \frac{3}{M_P^2}\hat{W}\bar{\hat{W}}\right) + \text{ D-terms}. \tag{4.12}$$

$\mathcal{K}$ is the Kähler potential, which has mass dimension 2, and $\hat{W}$ the superpotential, with mass dimension 3. M_P is the reduced Planck mass $M_P = \frac{1}{(8\pi G)^{\frac{1}{2}}} = 2.4 \times 10^{18}\text{GeV}$ and the Planck and string scales are related by

$$M_P^2 = \frac{4\pi\mathcal{V}_s^0}{g_s^2 l_s^2} \qquad \text{or} \qquad m_s = \frac{g_s}{\sqrt{4\pi\mathcal{V}_s^0}}M_P . \tag{4.13}$$

Here $\mathcal{V}_s^0 = \langle\mathcal{V}_s\rangle$ is the string-frame volume at the minimum. As shown in the appendix, including the α' and non-perturbative corrections, the Kähler and superpotentials are

$$\begin{aligned}\frac{\mathcal{K}}{M_P^2} &= -2\ln\left(\mathcal{V}_s + \frac{\xi g_s^{\frac{3}{2}}}{2e^{\frac{3\phi}{2}}}\right) - \ln(-i(\tau-\bar{\tau})) - \ln\left(-i\int_{CY}\Omega\wedge\bar{\Omega}\right),\\ \hat{W} &= \frac{g_s^{\frac{3}{2}}M_P^3}{\sqrt{4\pi}l_s^2}\left(\int_{CY}G_3\wedge\Omega + \sum A_i e^{\frac{-2\pi}{g_s}T_i}\right) \equiv \frac{g_s^{\frac{3}{2}}M_P^3}{\sqrt{4\pi}}W .\end{aligned} \tag{4.14}$$

Here $\xi = -\frac{\zeta(3)\chi(M)}{2(2\pi)^3}$ and $T_i = \tau_i + ib_i$, where $\tau_i = \int d^4x\sqrt{g}$ is a 4-cycle volume and b_i its axionic partner arising from the RR 4-form; these are good Kähler coordinates for IIB orientifold compactifications. The factors of g_s are unconventional and arise from our definition of the 10-dimensional Einstein frame (A.2) ($g_{\mu\nu,s} = e^{(\phi-\phi_0)/2}g_{\mu\nu,E}$). The advantage of this definition is that the resulting volumes are a true measure of 'largeness', and the stringy non-perturbative nature of the $e^{-\frac{a_i}{g_s}T_i}$ term is manifest.

To calculate scalar masses, we must express the potential (4.12) in terms of canonically normalised fields. The Kähler metric for the complex structure moduli is given by

$$G_{i\bar{j}} = \partial_i \partial_{\bar{j}} \mathcal{K}_{cs} = \partial_i \partial_{\bar{j}} \ln \left(-i \int_{CY} \Omega \wedge \bar{\Omega} \right) . \tag{4.15}$$

In the no-scale approximation, which holds to leading order, the potential for the complex structure moduli is

$$V = \frac{g_s^4 M_P^4}{8\pi (\mathcal{V}_s^0)^2} \int d^4x \sqrt{-g_E} e^{\mathcal{K}_{cs}} \left[G^{a\bar{b}} (D_a W)(D_{\bar{b}} \bar{W}) \right] , \tag{4.16}$$

where the sum runs over complex structure moduli only. The inverse of (4.15) is hard to make explicit, as to do so would require knowledge of all the Calabi-Yau periods. However, as (4.15) is independent of dilaton and Kähler moduli, this process will not introduce extra factors of $\mathcal{V}_s$ or g_s. Thus if we assume numerical factors to be $\mathcal{O}(1)$, we find

$$m_{cs}^2 = \mathcal{O}(1) \frac{g_s^4 N^2 M_P^2}{4\pi (\mathcal{V}_s^0)^2} , \tag{4.17}$$

where $N \sim \mathcal{O}(\sqrt{\frac{\chi}{24}})$ is a measure of the typical number of flux quanta and arises from the $D_a W$ terms. We therefore have

$$m_{cs} = \mathcal{O}(1) \frac{g_s N m_s}{\sqrt{\mathcal{V}_s^0}} . \tag{4.18}$$

As emphasised in [41], one requires a clear separation between Kaluza-Klein and complex structure masses to trust the supergravity analysis. We have

$$\frac{m_{cs}}{m_{KK}} \sim \frac{g_s N}{2\pi (\mathcal{V}_s^0)^{\frac{1}{3}}} . \tag{4.19}$$

At large volumes, this ratio is much less than one, which is reassuring.

For the concrete $\mathbb{P}^4_{[1,1,1,6,9]}$ example, the Kähler moduli may be treated more explicitly. It is nonetheless hard to normalise the fields canonically across the entirety of moduli space. However, to compute the spectrum we only need normalise the moduli at the physical minimum. It turns out (see appendix B) that the appropriately normalised fields are

$$\begin{aligned} \tau_5^c &= \sqrt{\frac{3}{2}} \frac{\tau_5}{\tau_5^0} M_P , \qquad & b_5^c &= \sqrt{\frac{3}{2}} \frac{b_5}{\tau_5^0} M_P , \\ \tau_4^c &= \sqrt{\frac{3}{4}} \frac{\tau_4}{(\tau_5^0)^{\frac{3}{4}} (\tau_4^0)^{\frac{1}{4}}} M_P , \qquad & b_4^c &= \sqrt{\frac{3}{4}} \frac{b_4}{(\tau_5^0)^{\frac{3}{4}} (\tau_4^0)^{\frac{1}{4}}} M_P . \end{aligned} \tag{4.20}$$

Here $\tau_5^0 = \langle \tau_5 \rangle$, etc. The bosonic mass matrix follows by taking the second derivatives of the scalar potential with respect to τ_i^c and b_i^c. In the vicinity of the large volume minimum, the scalar potential takes the form

$$V = g_s^4 M_P^4 \left(\frac{\lambda' \sqrt{\tau_4} e^{-2\frac{a_4\tau_4}{g_s}}}{\tau_5^{\frac{3}{2}}} + \frac{\mu'}{\tau_5^3} \tau_4 e^{-\frac{a_4\tau_4}{g_s}} \cos(\frac{a_4 b_4}{g_s}) + \frac{\nu'}{\tau_5^{\frac{9}{2}}} \right). \tag{4.21}$$

As discussed in [14] we have

$$\lambda' \sim \frac{a_4^2 |A_4|^2}{g_s^2}, \qquad \mu' \sim \frac{a_4 |A_4 W_0|}{g_s}, \qquad \text{and} \qquad \nu' \sim \xi |W_0|^2 .$$

The b_5 axion appears in terms suppressed by $e^{-a_5\tau_5}$ and has not been written explicitly. As $\tau_5^0 \gg 1$, it follows that this field is essentially massless. In terms of the canonical fields, the scalar potential (4.21) becomes

$$V = \frac{\lambda \sqrt{\tau_4^c \beta} e^{-2a_4\beta\tau_4^c}}{(\tau_5^0)^{\frac{3}{2}} (\tau_5^c)^{\frac{3}{2}}} + \frac{\mu \tau_4^c \beta e^{-a_4\beta\tau_4^c} \cos(a_4 \beta b_4)}{(\tau_5^0)^3 (\tau_5^c)^3} + \frac{\nu}{(\tau_5^0)^{\frac{9}{2}} (\tau_5^c)^{\frac{9}{2}}}, \tag{4.22}$$

where $\tau_4 = \beta g_s \tau_4^c$. The mass matrix is

$$\mathrm{d}^2 V = \begin{pmatrix} \frac{\partial^2 V}{\partial\tau_5^c \partial\tau_5^c} & \frac{\partial^2 V}{\partial\tau_5^c \partial\tau_4^c} & \frac{\partial^2 V}{\partial\tau_5^c \partial b_4^c} \\ \frac{\partial^2 V}{\partial\tau_4^c \partial\tau_5^c} & \frac{\partial^2 V}{\partial\tau_4^c \partial\tau_4^c} & \frac{\partial^2 V}{\partial\tau_4^c \partial b_4^c} \\ \frac{\partial^2 V}{\partial b_4^c \partial\tau_5^c} & \frac{\partial^2 V}{\partial b_4^c \partial\tau_4^c} & \frac{\partial^2 V}{\partial b_4^c \partial b_4^c} \end{pmatrix}. \tag{4.23}$$

At the minimum this mass matrix takes the schematic form

$$\frac{M_P^2 g_s^4}{(\mathcal{V}_s^0)^2} \begin{pmatrix} \frac{a}{\mathcal{V}_s^0} & \frac{b}{(\mathcal{V}_s^0)^{\frac{1}{2}} g_s} & 0 \\ \frac{b}{(\mathcal{V}_s^0)^{\frac{1}{2}} g_s} & \frac{c}{g_s^2} & 0 \\ 0 & 0 & d \end{pmatrix}.$$

Cross terms involving the axion decouple and we obtain

$$\begin{aligned} m_{\tau_5^c} &= \mathcal{O}(1) \frac{g_s^2 W_0}{\sqrt{4\pi} (\mathcal{V}_s^0)^{\frac{3}{2}}} M_P , \qquad & m_{b_5^c} &\sim \exp(-\tau_5^0) , \\ m_{\tau_4^c} &= \mathcal{O}(1) \frac{a_4 g_s W_0}{\sqrt{4\pi} \mathcal{V}_s^0} M_P , \qquad & m_{b_4^c} &= \mathcal{O}(1) \frac{a_4 g_s W_0}{\sqrt{4\pi} \mathcal{V}_s^0} M_P . \end{aligned} \tag{4.24}$$

The division of scales between the large modulus (τ_5) and the small modulus (τ_4) will turn out to be general (see section 4.6). The $\mathcal{O}(1)$ factors depend on the detailed geometry of the particular Calabi-Yau and are therefore not written explicitly, although given the Kähler potential their numerical computation is straightforward.

The τ_4 and b_4 moduli have masses similar to the dilaton and complex structure moduli. We may then worry that our above treatment was inconsistent, as we first integrated out complex structure moduli and only then considered Kähler moduli. However, as discussed in section 4.1, from the form of the full potential we see that the solution derived by first integrating out the complex structure moduli remains a minimum of the full potential.

4.3 Fermion masses

The fermions divide into the gravitino and the fermionic partners of the chiral superfields. The gravitino mass is given by

$$m_{\frac{3}{2}} = e^{\mathcal{K}/2}|\hat{W}| = M_P \frac{g_s^2 e^{\frac{\mathcal{K}_{cs}}{2}} |W_0|}{\mathcal{V}_s^0 \sqrt{4\pi}} . \tag{4.25}$$

with $\mathcal{K}$ and $\hat{W}$ given by (4.14).

We use expressions appropriate for a Minkowski minimum and so assume we have included lifting terms. The mass matrix for the other fermions is then $[M_\psi]_{ij}\bar{\psi}_{Li}\psi_{Lj}$, where $[M_\psi]_{ij} = \sum_{n=1}^4 [M_\psi^n]_{ij}$, with

$$\begin{aligned}
\left[M_\psi^1\right]_{ij} &= -M_P e^{\mathcal{K}/2}|\hat{W}| \left\{ \mathcal{K}_{ij} + \frac{1}{3}\mathcal{K}_i\mathcal{K}_j \right\} , \\
\left[M_\psi^2\right]_{ij} &= -M_P e^{\mathcal{K}/2}|\hat{W}| \left\{ \frac{\mathcal{K}_i\hat{W}_j + \mathcal{K}_j\hat{W}_i}{3\hat{W}} - 2\frac{\hat{W}_i\hat{W}_j}{3\hat{W}^2} \right\} , \\
\left[M_\psi^3\right]_{ij} &= -M_P e^{\mathcal{K}/2}\sqrt{\frac{\bar{\hat{W}}}{\hat{W}}}\hat{W}_{ij} , \\
\left[M_\psi^4\right]_{ij} &= M_P e^{\mathcal{G}/2}\mathcal{G}_l(\mathcal{G}^{-1})^l_k\mathcal{G}^k_{ij} ,
\end{aligned} \tag{4.26}$$

where $\mathcal{G} = \mathcal{K} + \ln(\hat{W}) + \ln(\bar{\hat{W}})$, $\hat{W}_i = \partial_i\hat{W}$, $\mathcal{K}_i = \partial_i\mathcal{K}$, $\mathcal{G}^l = \partial_{\bar{l}}\mathcal{G}$, etc. Here derivatives are with respect to the canonically normalised fields (4.20).

There is one massless fermion corresponding to the goldstino, which is eaten by the gravitino. Essentially this is $\tilde{\tau}_5$, although there is some small mixing with $\tilde{\tau}_4$. The mass of $\tilde{\tau}_4$ can be calculated from (4.26); we find

$$m_{\tilde{\tau}_4} \approx \frac{g_s^2 a_4 W_0}{\mathcal{V}_s} M_P \approx m_{\tau_4} \approx m_{3/2} . \tag{4.27}$$

As with the bosonic spectrum, it is hard to obtain explicit expressions for modulino masses for the complex structure moduli. However, there is no explicit volume dependence in $\mathcal{K}_{cs}$ or W_{flux}, and so the volume dependence of $m_{\tilde{\phi}}$ is determined by the $e^{\frac{\mathcal{K}}{2}}$ terms. Therefore

$$m_{\tilde{\phi}} \sim \frac{g_s^2 W_0}{\mathcal{V}_s} M_P , \qquad m_{\tilde{\tau}} \sim \frac{g_s^2 W_0}{\mathcal{V}} M_P . \tag{4.28}$$

and modulino masses have a scale set by the gravitino mass. Thus, as expected, $m_{3/2}$ determines the scale of Bose-Fermi splitting. As at large volume $m_{3/2} \ll m_s, m_{KK}$, the moduli and modulino physics should decouple from that associated with stringy or Kaluza-Klein modes.

4.4 Moduli spectroscopy

At large volume, the single most important factor determining the moduli masses is the stabilised internal volume. The different scales are suppressed compared to the 4-dimensional

Scale	Mass
4-dimensional Planck mass	$\frac{4\pi\mathcal{V}_s^0}{g_s} m_s = M_P$
String scale m_s	$m_s = \frac{g_s}{\sqrt{4\pi\mathcal{V}_s^0}} M_P$
Stringy modes m_S	$2\pi m_s = \frac{g_s\sqrt{\pi}}{\sqrt{\mathcal{V}_s^0}} M_P$
Kaluza-Klein modes m_{KK}	$\frac{2\pi}{\mathcal{V}_s^{\frac{1}{6}}} m_s = \frac{g_s\sqrt{\pi}}{(\mathcal{V}_s^0)^{\frac{2}{3}}} M_P$
Gravitino $m_{3/2}$	$\frac{g_s W_0}{\sqrt{\mathcal{V}_s^0}} m_s = \frac{g_s^2 W_0}{\sqrt{4\pi}\mathcal{V}_s^0} M_P$
Dilaton-axion m_τ	$\frac{g_s N}{\sqrt{\mathcal{V}_s^0}} m_s = \frac{g_s^2 N}{\sqrt{4\pi}\mathcal{V}_s^0} M_P$
Complex structure moduli m_ϕ	$\frac{g_s N}{\sqrt{\mathcal{V}_s^0}} m_s = \frac{g_s^2 N}{\sqrt{4\pi}\mathcal{V}_s^0} M_P$
'Small' Kähler modulus $m_{\tau_4}, m_{\tilde{\tau}_4}$	$\frac{W_0}{\sqrt{\mathcal{V}_s^0}} m_s = \frac{g_s W_0}{\sqrt{4\pi}\mathcal{V}_s^0} M_P$
Modulinos $m_{\tilde{\tau}}, m_{\tilde{\phi}}$	$\frac{g_s W_0}{\sqrt{\mathcal{V}_s^0}} m_s = \frac{g_s^2 W_0}{\sqrt{4\pi}\mathcal{V}_s^0} M_P$
'Large' volume modulus m_{τ_5}	$\frac{g_s W_0}{\mathcal{V}_s^0} m_s = \frac{g_s^2 W_0}{\sqrt{4\pi}(\mathcal{V}_s^0)^{\frac{3}{2}}} M_P$
Volume axion m_{b_5}	$\exp(-(\mathcal{V}_s^0)^{\frac{2}{3}}) M_P \sim 0$

Table 1: Moduli spectrum for $\mathbb{P}^4_{[1,1,1,6,9]}$ in terms of $\mathcal{V}_s^0 = \langle \mathcal{V}_s \rangle$.

Planck scale by various powers of the internal volume. In table 1 we show this scaling explicitly for the various moduli. There are also model-dependent $\mathcal{O}(1)$ factors, which we do not show explicitly.

This spectrum has several characteristic features. First, the string scale is hierarchically smaller than the Planck scale. The internal volume depends exponentially on the inverse string coupling and thus very small changes in the stabilised dilaton lead to large effects in the compact space. A standard criticism of the large extra dimensions scenario was that it seemed very difficult to obtain naturally from string theory; the above shows that this criticism does not hold here.

The majority of moduli masses are stabilised at a high scale $\mathcal{O}\left(\frac{M_P}{\mathcal{V}_s^0}\right)$, comparable to $m_{\frac{3}{2}}$ but below the scale of stringy and Kaluza-Klein modes. There are two light moduli; the radion and its associated axion. The latter has an extremely small mass that is light in any units. As an axion, one would like to use this as a solution to the strong CP problem. Unfortunately this axion corresponds precisely to the D7 gauge theory with gauge coupling determined by the inverse size of the large 4-cycle which then would be extremely small, and so we do not expect the Standard Model to live on such branes. The radion may have cosmological implications which it would be interesting to explore further.

The principal factor entering the scales is the internal volume, and we present in table 2 possible spectra arising from various choices of the internal volume. We would like to emphasise that the reason we can talk about 'choices' of the volume is that its stabilised value is exponentially sensitive to $\mathcal{O}(1)$ parameters such as the string coupling, and thus it may be dialled freely from the Planck to TeV scales. The moduli spectra are shown for GUT, intermediate and TeV string scales. If the fundamental scale is at the GUT scale, all moduli are heavy with the exception of the light b_5 axion. For the intermediate scale,

Scale	Mass	GUT	Intermediate	TeV
M_P	M_P	$2.4\times10^{18}\,\text{GeV}$	$2.4\times10^{18}\,\text{GeV}$	$2.4\times10^{18}\,\text{GeV}$
m_s	$\frac{g_s}{\sqrt{4\pi\mathcal{V}_s^0}}M_P$	$1.0\times10^{15}\,\text{GeV}$	$1.0\times10^{12}\,\text{GeV}$	$1.0\times10^{3}\,\text{GeV}$
m_S	$2\pi m_s=\frac{g_s\sqrt{\pi}}{\sqrt{\mathcal{V}_s^0}}M_P$	$6\times10^{15}\,\text{GeV}$	$6\times10^{12}\,\text{GeV}$	$6\times10^{3}\,\text{GeV}$
m_{KK}	$\frac{2\pi m_s}{(\mathcal{V}_s^0)^{\frac{1}{6}}}=\frac{g_s\sqrt{\pi}}{(\mathcal{V}_s^0)^{\frac{2}{3}}}M_P$	$1.5\times10^{15}\,\text{GeV}$	$1.5\times10^{11}\,\text{GeV}$	$0.15\,\text{GeV}$
$m_{3/2}$	$\frac{g_s^2W_0}{\sqrt{4\pi\mathcal{V}_s^0}}M_P$	$1.5\times10^{12}\,\text{GeV}$	$1.5\times10^{6}\text{GeV}$	$1.5\times10^{-12}\text{GeV}$
m_τ	$\frac{g_sNm_s}{\sqrt{\mathcal{V}_s^0}}=\frac{g_s^2N}{\sqrt{4\pi\mathcal{V}_s^0}}M_P$	$1.5\times10^{12}\,\text{GeV}$	$1.5\times10^{6}\text{GeV}$	$1.5\times10^{-12}\text{GeV}$
m_{cs}	$\frac{g_sNm_s}{\sqrt{\mathcal{V}_s^0}}=\frac{g_s^2N}{\sqrt{4\pi\mathcal{V}_s^0}}M_P$	$1.5\times10^{12}\,\text{GeV}$	$1.5\times10^{6}\text{GeV}$	$1.5\times10^{-12}\text{GeV}$
m_{τ_4}, m_{b_4}	$\frac{a_4g_sW_0}{\sqrt{4\pi\mathcal{V}_s^0}}M_P$	$1.5\times10^{11}\,\text{GeV}$	$1.5\times10^{5}\text{GeV}$	$1.5\times10^{-11}\text{GeV}$
m_{τ_5}	$\frac{g_s^2W_0}{\sqrt{4\pi}(\mathcal{V}_s^0)^{\frac{3}{2}}}M_P$	$2.2\times10^{10}\,\text{GeV}$	$22\,\text{GeV}$	$2.2\times10^{-26}\,\text{GeV}$
m_{b_5}	$\exp(-a_5\tau_5)M_P\sim0$	$\sim10^{-300}\,\text{GeV}$	$\exp(-10^6)\,\text{GeV}$	$\exp(-10^{18})\,\text{GeV}$

Table 2: Moduli spectra for GUT, intermediate and TeV string scales.

the volume modulus τ_5 is relatively light. Its mass does not present a problem with fifth force experiments but may be problematic in a cosmological context [43, 44]. Finally, for TeV scale gravity all moduli are very light. In particular, τ_5 is now so light ($10^{-17}\,\text{eV}$) to be in conflict with fifth force experiments. It would then be difficult to realise this scenario unless either for some reason observable matter did not couple to τ_5 or its mass received extra corrections.

4.5 On the validity of the effective field theory

Using these results we can go back and check that the four-dimensional effective field theory is self-consistent. Our use of an $\mathcal{N}=1$ supergravity framework should be valid so long as there is a separation of scales in which the 4-dimensional physics decouples from the high-energy physics. Let us enumerate the consistency conditions that a candidate minimum must satisfy in order to trust the formalism. First,

$$\langle\mathcal{V}_s\rangle=\langle\mathcal{V}_E g_s^{\frac{3}{2}}\rangle\gg1\,. \tag{4.29}$$

To control the α' expansion, the string-frame compactification volume must be much greater than unity. It is important to keep track of frames here; this condition is often incorrectly stated with an Einstein frame ($g_{\mu\nu,s}=e^{\phi/2}g_{\mu\nu,E}$) volume. However, in Einstein frame the α' corrections come with inverse powers of g_s (as in (3.9)) and thus the string frame volume is the correct measure.

Secondly, we require

$$\begin{aligned}\langle V\rangle&\ll m_s^4\,,\\ \langle V\rangle&\ll m_{KK}^4\,.\end{aligned} \tag{4.30}$$

The vacuum energy must be suppressed compared to the string and compactification scales. Otherwise, the neglect of stringy and Kaluza-Klein modes in the analysis is untrustworthy.

We likewise require the particle masses to decouple, namely that

$$m_{\frac{3}{2}},\ m_{\phi},\ m_{\tilde{\phi}} \ll m_s,\ m_{KK}\,, \tag{4.31}$$

where ϕ and $\tilde{\phi}$ are generic moduli and modulini.

There is one further potential constraint we wish to discuss. The $\mathcal{N} = 1$ SUGRA energy can be written

$$V_{\mathcal{N}=1} = \underbrace{e^{\mathcal{K}}\left[G^{i\bar{j}}D_iWD_{\bar{j}}\bar{W}\right]}_{\text{F-term energy}} - \underbrace{3e^{\mathcal{K}}|W|^2}_{\text{gravitational energy}}\,, \tag{4.32}$$

and we may thus define a susy breaking energy $m_{\rm susy}$, by $m^4_{\rm susy} = e^{\mathcal{K}}\left[G^{i\bar{j}}D_iWD_{\bar{j}}\bar{W}\right]$. In no-scale models, $m^4_{\rm susy}$ cancels against $3e^{\mathcal{K}}|W|^2$ to give a vanishing cosmological constant. Is it necessary to require $m_{\rm susy} \ll m_{KK}, m_s$? As $m_{\rm susy} \sim W_0 m_s$, imposing this would lead to the constraints

$$\begin{aligned} W_0 &\ll 1 \\ W_0\langle\mathcal{V}_s\rangle^{\frac{1}{3}} &\ll 1\,. \end{aligned} \tag{4.33}$$

Normally, there is a twofold reason for requiring $m_{\rm susy} \ll m_s$. First, once susy is broken the vacuum energy is of $\mathcal{O}(m^4_{\rm susy})$ and secondly, the boson-fermion mass splittings are $\mathcal{O}(m_{\rm susy})$. However, neither reasons are valid here. In a no-scale model, irrespective of the value of $m_{\rm susy}$, the vacuum energy vanishes. Furthermore, the boson associated with supersymmetry breaking is massless in the no-scale approximation (as the potential is flat), whereas the associated fermion is the goldstino that is eaten by the gravitino. It is the gravitino that sets the scale of Bose-Fermi splitting, but this has mass $m_{3/2} = e^{\frac{\mathcal{K}}{2}}|W| \sim \frac{m_{\rm susy}}{\sqrt{\mathcal{V}}}$ at large volumes. Requiring $m_{3/2} \ll m_{KK}, m_s$ leads to the much weaker constraint

$$W_0 \ll \langle\mathcal{V}_s\rangle^{\frac{1}{3}}\,. \tag{4.34}$$

Thus $m_{\rm susy}$ as defined above is an imaginary scale; it sets neither the scale of Bose-Fermi splitting nor the vacuum energy. Thus we shall only consider (4.29), (4.30) and (4.31) as relevant constraints. We should note that this distinction only arises because we are in a large extra dimensions scenario. If $\langle\mathcal{V}_s\rangle \sim \mathcal{O}(1)$, the conditions (4.33) and (4.34) coalesce.

There is finally a consistency conditions peculiar to IIB flux compactifications. If τ_i are the divisor volumes, we require

$$\frac{a_i\tau_{i,s}}{g_s} = a_i\tau_{i,E} \gg 1\,. \tag{4.35}$$

This allows us to neglect multi-instanton contributions.

Let us now consider these constraints as applied to our model. As the stabilised volume is exponentially large, (4.29) is trivially satisfied. The conditions (4.30) depend on the vacuum energy at the minimum. After the de Sitter uplift, these are satisfied by construction. Before the uplift, we recall that the vacuum energy at the minimum is $\mathcal{O}(\frac{W_0^2 M_P^4 g_s^4}{4\pi \mathcal{V}^3})$, whereas $m_{KK}^4 \sim \frac{\pi^2 g_s^4 M_P^4}{\mathcal{V}^{\frac{8}{3}}}$. This gives a restriction

$$W_0 \ll \pi^{\frac{3}{2}} \langle \mathcal{V}_s \rangle^{\frac{1}{6}} . \tag{4.36}$$

At large volumes this is not an onerous condition to satisfy.

The particle mass constraints are likewise satisfied. The most dangerous of these is the requirement $m_{3/2}, m_\phi \ll m_{KK}$. From (4.19) we require

$$\frac{g_s N}{2\pi} \ll \langle \mathcal{V} \rangle^{\frac{1}{3}} , \tag{4.37}$$

where N is a measure of the typical number of flux quanta[8] and can be taken to be $\mathcal{O}(\sqrt{\chi/24}) \sim 30$. At the large volumes we work at this constraint is satisfied comfortably. The constraints on the divisor volumes are also satisfied at large volume, as for the 'small' divisor τ_4, $a_4 \tau_4 \sim \ln \mathcal{V}$ at the minimum.

Thus all consistency conditions are satisfied and we see no reason to regard the use of $\mathcal{N} = 1$ supergravity as inconsistent. An important point is that we obtain no strong constraints on the value of W_0; this is in contrast to KKLT-type solutions, for which very small values of W_0 are essential. As large values of W_0 are preferred by the statistical results of Douglas and collaborators [24, 12, 23], we expect a typical solution to have large W_0. The maximum value of W_0 is determined by the fluxes satisfying the tadpole conditions and can be in general of order $10 - 100$.

In KKLT constructions the constraint (4.29) leads to the requirement $W_0 \ll 1$ and g_s not too small. To be more precise, as $A_i e^{-\frac{a_i \tau_i}{g_s}} \sim W_0$, this gives

$$-\frac{g_s \ln(W_0)}{a_i} \gg 1 .$$

If this is satisfied then satisfying (4.30) is automatic. As in such compactifications $\langle \mathcal{V}_s \rangle^{\frac{1}{3}} \sim \mathcal{O}(1)$, (4.31) gives

$$\frac{g_s N}{2\pi} \ll \mathcal{O}(1) .$$

This is in general hard to satisfy (as pointed out in [41]).

4.6 The general case

We have so far focused on a particular model, $\mathbb{P}^4_{[1,1,1,6,9]}$. Let us now argue that the above framework will in fact be valid for all Calabi-Yaus with $h^{2,1} > h^{1,1}$, assuming the existence of appropriate non-perturbative superpotentials.

In the $\mathbb{P}^4_{[1,1,1,6,9]}$ example, we found that of the two Kähler moduli τ_4, τ_5 one (τ_5) was large, whereas the other (τ_4) was stabilised at a small value. Our claim is that this behaviour will persist in the general case, with only one Kähler modulus large and all others tending to be small. Therefore there is only one modulus responsible for the large volumes obtained.

[8] Not to be confused with N measuring the rank of the hidden sector group that enters in the coefficients a_i for gaugino condensation potentials.

To argue this, let us write the large-volume expression for the scalar potential as:

$$V = \frac{C_1 e^{-a_i\tau_i - a_j\tau_j}}{\mathcal{V}} - \frac{C_2 e^{-a_i\tau_i}}{\mathcal{V}^2} + \frac{C_3}{\mathcal{V}^3}\,, \tag{4.38}$$

with the factors C_i depending on powers of the Kähler moduli at the most. $C_3 \sim -\chi(M)$ and so it is positive as long as $h^{2,1} > h^{1,1}$. We shall not quibble here over frames or the factors of g_s in the exponent as they do not affect the argument.

Let us start in a position where $\mathcal{V} \gg 1$, $V < 0$ and there are many large moduli $\tau_i \gg 1$. We may investigate the behaviour of the potential as one of the τ_i fields change. Originally, all terms non-perturbative in the large τ_i can be neglected. As the second term is the only negative term, it wants to increase its magnitude in order to minimise the value of the potential. This will naturally reduce the value of the corresponding τ_i in order for the exponential to be more relevant. As long as the other large moduli are adjusted to keep $\mathcal{V}$ constant, this reduces the value of the potential. This continues until $e^{-a_i\tau_i} \sim \frac{1}{\mathcal{V}}$, when the (positive) double exponential term in (4.38) becomes important and the modulus will be stabilised.

We may carry on doing this with all the large moduli, but since we are holding $\mathcal{V}$ constant, one (combination) of the fields must remain large. This finally leaves us with a picture of a manifold with one large 4-cycle (and corresponding 2-cycle), but all other cycles of size close to the string scale.

In order to get a clearer picture let us make a brief geometric digression. The volume of a Calabi-Yau can be expressed either in terms of 2-cycles, $\mathcal{V} = \frac{1}{6}k_{ijk}t^i t^j t^k$, or 4-cycles $\mathcal{V} = \mathcal{V}(\tau_i)$. Then the matrix

$$M_{ij} = \frac{\partial^2 \mathcal{V}}{\partial\tau_i \partial\tau_j} \tag{4.39}$$

has signature $(1, h^{1,1} - 1)$ (one plus, the rest minus). This follows from the result that $\tau_i = \tau_i(t^j)$ is simply a coordinate change on Kähler moduli space, and it is a standard result [45] that

$$M'_{ij} = \frac{\partial^2 \mathcal{V}}{\partial t^i \partial t^j} \tag{4.40}$$

has signature $(1, h^{1,1} - 1)$. The coordinate change cannot change the signature of the metric.

This signature manifests itself in explicit models. In both the $\mathbb{P}^4_{[1,1,1,6,9]}$ example studied above and an $\mathcal{F}_{11}$ model also studied in [17], the volume may in fact be written explicitly in terms of the divisor volumes. With the τ_i as defined in [17],[9] we have

$$\begin{aligned}
\mathbb{P}^4_{[1,1,1,6,9]} \quad \mathcal{V} &= \frac{1}{9\sqrt{2}}\left(\tau_5^{\frac{3}{2}} - \tau_4^{\frac{3}{2}}\right),\\
\tau_4 &= \frac{t_1^2}{2} \qquad \text{and} \qquad \tau_5 = \frac{(t_1 + 6t_5)^2}{2}\,.\\
\mathcal{F}_{11} \quad \mathcal{V} &= \frac{1}{3\sqrt{2}} 2\left(\tau_1 + \tau_2 + 2\tau_3\right)^{3/2} - \left(\tau_2 + 2\tau_3\right)^{3/2} - \tau_2^{3/2}\,,\\
\tau_1 &= \frac{t_2}{2}\left(2t_1 + t_2 + 4t_3\right), \qquad \tau_2 = \frac{t_1^2}{2}\,, \qquad \text{and} \qquad \tau_3 = t_3\left(t_1 + t_3\right).
\end{aligned} \tag{4.41}$$

[9]Note that here the $\tau_i \neq \frac{\partial \mathcal{V}}{\partial t_i}$, but are rather linear combinations thereof.

JHEP08(2005)007

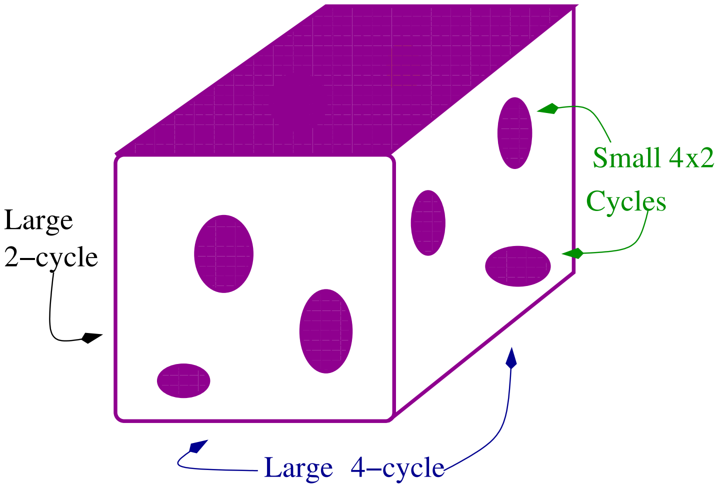

Figure 2: A Swiss cheese picture of a Calabi-Yau. There is one pair of large 2- and 4-cycles — increasing the cycle volume increases the overall volume. The other pairs are such that increasing the cycle volume decreases the overall volume.

For the $\mathcal{F}_{11}$ model, from the expressions for τ_i in terms of 2-cycles, we may see that it is consistent to have τ_1 large and τ_2, τ_3 small but not otherwise. The signature of d^2V is manifest in (4.41); each expression contains $h^{1,1}-1$ minus signs. There is another important point. In each case, there is a well-defined limit in which the overall volume goes to infinity and all but one divisors remain small. These limits are given by ($\tau_5 \to \infty, \tau_4$ constant) and ($\tau_1 \to \infty, \tau_2, \tau_3$ constant) respectively. Furthermore, in each case this limit is unique: e.g. the alternative limit ($\tau_2 \to \infty, \tau_1, \tau_3$ constant) is not well-defined.

This motivates a 'Swiss-cheese' picture of the Calabi-Yau, illustrated in figure 2. A Swiss cheese is a 3-manifold with 2-cycles. Of these 2-cycles, one (t_b) is 'large' and the others ($t_{s,i}$) are small. The volume of the cheese can be written

$$\mathcal{V} = t_b^{3/2} - \sum_i t_{s,i}^{3/2}\,, \tag{4.42}$$

and $\frac{\partial^2 \mathcal{V}}{\partial t_i \partial t_j}$ has signature $(1, h^2 - 1)$. The small cycles are internal; increasing their volume decreases the overall volume of the manifold. There is one distinguished cycle that controls the overall volume; this cycle may be made arbitrarily large while holding all other cycles small, and controls the overall volume. For all other cycles, an arbitrary increase in their volume decreases the overall volume and eventually leads to an inconsistency. The small cycles may be thought of as local effects; if the bulk cycle is large, the overall volume is largely insensitive to the size of the small cycles.

To capture the above, let us consider a Calabi-Yau with divisors $\tau_b, \tau_{s,i}$ such that the volume can be written

$$\mathcal{V} = \left(\tau_b + \sum a_i \tau_{s,i}\right)^{\frac{3}{2}} - \left(\sum b_i \tau_{s,i}\right)^{\frac{3}{2}} - \cdots - \left(\sum k_i \tau_{s,i}\right)^{\frac{3}{2}}. \tag{4.43}$$

We assume that a limit $\tau_b \gg \tau_{s,i}$ is well-defined. By working in this limit, the minus signs can be seen to follow from (4.39). The form given above is valid globally for both $\mathbb{P}^4_{[1,1,1,6,9]}$

JHEP08(2005)007

and $\mathcal{F}_{11}$ models. The form (4.43) is illustrative and it is not important for our argument that it hold generally; the important assumption is that there exists a well-defined limit $\tau_b \gg \tau_{s,i}$. We also note that the argument that follows can then be recast as in [14] using expressions solely in terms of 2-cycle moduli. In the limit $\tau_b \to \infty, \tau_{s,i}$ small , the scalar potential takes the form

$$V = e^{\mathcal{K}} \left[G^{i\bar{j}} \partial_i W \partial_{\bar{j}} \bar{W} + G^{i\bar{j}}((\partial_i \mathcal{K}) W \partial_{\bar{j}} \bar{W} + c.c.) + \frac{3\xi}{4\mathcal{V}} \right]. \tag{4.44}$$

We take W to be

$$W = W_0 + \sum_{s,i} A_i e^{-a_i T_i}. \tag{4.45}$$

We may include an exponential dependence on τ_b in W; as $\tau_b \gg 1$ this is in any case insignificant. Now,

$$G^{i\bar{j}} \partial_i \mathcal{K} \propto \tau_j , \tag{4.46}$$

and we may also verify that, so long as i and j both correspond to small moduli,

$$G^{i\bar{j}} \sim \mathcal{V} \sqrt{\tau_s'} \tag{4.47}$$

where τ_s' is a complicated function of the $\tau_{s,i}$ that scales linearly under $\tau_{s,i} \to \lambda \tau_{s,i}$. The scalar potential then takes the form

$$V = \frac{\sqrt{\tau_s'} \partial_i W \cdot \partial_j W}{\mathcal{V}} - \frac{\tau_i \cdot \partial_i W}{\mathcal{V}^2} + \frac{3\xi}{4\mathcal{V}^3}. \tag{4.48}$$

If we then take the decompactification limit $\tau_b \to \infty$ with $\partial_i W = \frac{1}{\mathcal{V}}$ and $\tau_i \sim \mathcal{O}(\ln \mathcal{V})$, then the potential goes to zero from below. As this result is independent of the strength of the non-perturbative corrections, which always eventually dominate the positive α'^3 terms, there is an associated minimum at large volume.

The geometric interpretation of this is that the 'external' cycle controlling the overall volume may be very large, whereas the small, 'internal' cycles are always stabilised at small volumes. As in section 4.2, the masses associated with the moduli parametrising the small cycles appear at a high scale with $m_{\tau_i} \sim \mathcal{O}(\frac{g_s^2 W_0}{\mathcal{V}})$, comparable to the masses of the dilaton and complex structure moduli. Thus the resulting spectrum is largely model-independent — the moduli associated with small internal cycles take masses at a high scale and decouple from the low energy physics. However, the 'volume' modulus, which is distinguished and model-independent, is relatively light and may be of cosmological importance.

Even though we think we have explored a generic case there is still room for different pictures to emerge. In particular, we may imagine a Calabi-Yau for which $\mathcal{V} = t_1 t_2^2 + F(t_n)$ with $t_n \neq t_1, t_2$. Then $\tau_1 = t_2^2$ and $\tau_2 = 2t_1 t_2$, and so if we had τ_2 large and τ_1 small, we would have t_1 large and t_2 small. This will give a throat-like picture where one throat has a large two-cycle and small four-cycle whereas the other throat has a large four-cycle and a small two-cycle, with all other cycles small. This would give the interesting possibility of making contact with the 0.1 mm fundamental scale scenario since then $\mathcal{V} \sim t_1$ and, in the throat, there are only two large dimensions [16, 42]. This may be of phenomenological interest given the potential to search for deviations of gravity at the submillimeter scale and the connection between this scale and dark energy [42]. However, we do not know if there are Calabi-Yau manifolds with this property.

Another possibility would be the existence of Calabi-Yaus for which the limit $\tau_b \gg \tau_{s,i}$ is not well-defined. In that case, it would not be possible to realise large volumes without having several large divisors rather than just one. This does not hold for $\mathbb{P}^4_{[1,1,1,6,9]}$ or $\mathcal{F}_{11}$; if it were to hold for other models it would be interesting to study the consequences for the above mechanism.

5. Soft supersymmetry breaking terms

We have so far described the spectrum of the closed moduli sector of the theory. In a typical string model we will have, besides these fields, the open string moduli, usually corresponding to the location of D-branes, and matter fields living on D-branes. For phenomenological purposes, it is this spectrum that has more relevance. In this section we will study the soft supersymmetry breaking lagrangian corresponding to the large volume minimum we have found.

It is worth noticing that this sector is more model dependent and depends on how the standard model is embedded within this string theory construction: for a given Calabi-Yau flux compactification we may have several different ways to embed the standard model.

Let us concentrate on the two main possibilities by considering matter fields coming from D3 or D7 branes.

5.1 The moduli-matter couplings

In [6] the general Kähler potential for Calabi-Yau orientifolds with D3 branes was derived by a dimensional reduction. The result is

$$K(S,T,U,\phi) = -\log(S+\bar{S}) - \log(-i\int \Omega\wedge\bar{\Omega}) - 2\log(\mathcal{V}(T,U,\phi))\,. \tag{5.1}$$

Here U are the complex structure moduli and $\phi^i, i = 1,2,3$ are the scalar fields corresponding to the position of a stack of D3 branes on the Calabi-Yau. The Calabi-Yau volume $\mathcal{V}$ is to be understood as a function of the complexified Kähler moduli T, the expression for which is:

$$T_\alpha = \tau_\alpha + i\rho_\alpha + i\mu_3 l^2 (\omega_\alpha)_{i\bar{j}}\,\mathrm{Tr}\,\phi^i\left(\bar{\phi}^{\bar{j}} - \frac{i}{2}\bar{U}^{\hat{a}}(\bar{\chi}_{\hat{a}})^{\bar{j}}_{l}\phi^l\right). \tag{5.2}$$

ω_α are a basis for (1,1) forms on the Calabi-Yau which survive the orientifold projection, while $\chi_{\hat{a}}$ form a basis for $(2,1)$ forms that have negative sign under the orientifold projection. Their number equals the number of complex structure moduli $U^{\hat{a}}$ surviving the orientifold projection. $\mu_3 = (2\pi)^{-3}\alpha'^{-2}$ is the RR charge of a single D3 brane. Also $l = 2\pi\alpha'$ and so $\mu_3 l^2 = 1/(2\pi)$. For orientifold compactifications with both D3 and D7 branes, the Kähler potential was derived in [46] and has the form

$$\mathcal{K}(S,T,U,\zeta) = -\log(S+\bar{S}+2i\mu_7\mathcal{L}_{AB}\zeta^A\bar{\zeta}^{\bar{B}}) - \log(-i\int\Omega\wedge\bar{\Omega}) - 2\log(\mathcal{V}(T,U,\phi))\,, \tag{5.3}$$

JHEP08(2005)007

where $\mathcal{L}_{AB}$ are certain geometric quantities and ζ^A are the moduli describing the position of the D7 brane. We are neglecting for now the possibility of Wilson line moduli.

5.2 F-terms

We will use the standard formalism for calculating soft supersymmetry breaking terms, as described for example in [47]. We proceed by expanding the Kähler potential and superpotential in terms of the visible sector fields φ

$$
\begin{aligned}
\mathcal{K} &= \hat{\mathcal{K}} + (\tilde{\mathcal{K}}_{i\bar{j}})\varphi^i\bar{\varphi}^{\bar{j}} + Z_{ij}\varphi^i\varphi^j + \cdots\,, \\
W &= \hat{W} + \mu_{ij}\varphi^i\varphi^j + Y_{ijk}\varphi^i\varphi^j\varphi^k + \cdots\,,
\end{aligned} \tag{5.4}
$$

where $\hat{\mathcal{K}}, \tilde{\mathcal{K}}_{i\bar{j}}, Z_{ij}, \mu_{ij}$ and Y_{ijk} depend on the hidden moduli only. F-term supersymmetry breaking in a hidden sector is characterised by nonvanishing expectation values for the auxiliary fields of the hidden sector chiral superfields. These F-terms may be written as

$$
\bar{F}^{\bar{m}} = e^{\hat{\mathcal{K}}/2M_P^2}\hat{\mathcal{K}}^{\bar{m}n}\frac{D_n\hat{W}}{M_P^2}\,. \tag{5.5}
$$

In this and all subsequent formulae, m and n range over the hidden moduli — the dilaton, complex structure and Kähler moduli in our case. We henceforth work in Planck mass units and do not include explicit factors of M_P.

Given the F-terms, the various soft parameters can be calculated. For example, assuming a diagonal matter field metric $\tilde{\mathcal{K}}_{i\bar{j}} = \delta_{i\bar{j}}\tilde{\mathcal{K}}_i$,[10] the masses squared of canonically normalised matter fields φ^i can be written as

$$
m_i^2 = m_{3/2}^2 + V_0 - F^m\bar{F}^{\bar{n}}\partial_m\partial_{\bar{n}}\log\tilde{\mathcal{K}}_i\,, \tag{5.6}
$$

where V_0 denotes the value of the cosmological constant. The normalised gaugino masses for a sector with gauge kinetic function f_a are

$$
M_a = \frac{1}{2}(\mathrm{Re}\, f_a)^{-1}F^m\partial_m f_a\,, \tag{5.7}
$$

while the A-terms of normalised matter fields $\hat{\varphi}^i$ are $A_{ijk}\hat{Y}_{ijk}\hat{\varphi}^i\hat{\varphi}^j\hat{\varphi}^k$ with

$$
\begin{aligned}
A_{ijk} &= F^m(\hat{\mathcal{K}}_m + \partial_m\log Y_{ijk} - \partial_m\log(\tilde{\mathcal{K}}_i\tilde{\mathcal{K}}_i\tilde{\mathcal{K}}_k))\,, \\
\hat{Y}_{ijk} &= Y_{ijk}\frac{\hat{W}^*}{|\hat{W}|}e^{\hat{\mathcal{K}}/2}(\tilde{\mathcal{K}}_i\tilde{\mathcal{K}}_j\tilde{\mathcal{K}}_k)^{-1/2}\,.
\end{aligned} \tag{5.8}
$$

Finally, if $Z_{ij} = \delta_{ij}Z$ and $\mu_{ij} = \mu\delta_{ij}$, the B-term $\hat{\mu}B\hat{\varphi}^i\hat{\varphi}^i$ for the field φ^i can be written as

$$
\begin{aligned}
\hat{\mu}B = (\tilde{\mathcal{K}}_i)^{-1}\Bigg\{ &\frac{\hat{W}^*}{|\hat{W}|}e^{\hat{\mathcal{K}}/2}\mu\Big[F^m(\hat{\mathcal{K}}_m + \partial_m\log\mu - 2\partial_m\log(\tilde{\mathcal{K}}_i)) - m_{3/2}\Big] + \\
&+ (2m_{3/2}^2 + V_0)Z - m_{3/2}\bar{F}^{\bar{m}}\partial_{\bar{m}}Z + \\
&+ m_{3/2}F^m(\partial_m Z - 2Z\partial_m\log(\tilde{\mathcal{K}}_i)) - \\
&- \bar{F}^{\bar{m}}F^n(\partial_{\bar{m}}\partial_n Z - 2\partial_{\bar{m}}Z\partial_n\log(\tilde{\mathcal{K}}_i))\Bigg\}\,.
\end{aligned} \tag{5.9}
$$

[10] $\tilde{\mathcal{K}}_i$ is not to be confused with the derivative of the Kähler potential with respect to modulus i.

where the effective μ-term is given by

$$\hat{\mu} = \left(\frac{\hat{W}^*}{|\hat{W}|} e^{\hat{\mathcal{K}}/2} \mu + m_{3/2} Z - F^{\bar{m}} \partial_{\bar{m}} Z \right) (\tilde{\mathcal{K}}_i)^{-1} \,. \tag{5.10}$$

We shall set $V_0 = 0$ since soft masses are to be evaluated after lifting the vacuum energy.

5.2.1 Relative size of F-terms

It is important to have an idea of the approximate sizes of the various F-terms used in the computation of soft terms. We shall again work in the context of the $\mathbb{P}^4_{[1,1,1,6,9]}$ model with two Kähler moduli T_4, T_5. At large volume, the relevant parts of the inverse metric are

$$\begin{aligned} \mathcal{K}^{\bar{T}_4 T_4} &\sim \mathcal{V}\,, \\ \mathcal{K}^{\bar{T}_4 T_5} &\sim \mathcal{V}^{2/3}\,, \\ \mathcal{K}^{\bar{T}_5 T_5} &\sim \mathcal{V}^{4/3}\,, \\ \mathcal{K}^{\bar{S} T_4} &\sim \frac{1}{\mathcal{V}}\,, \\ \mathcal{K}^{\bar{S} T_5} &\sim \frac{1}{\mathcal{V}^{1/3}}\,. \end{aligned} \tag{5.11}$$

The derivatives of the Kähler potential are $\partial_4 \mathcal{K} \equiv \partial_{T_4} \mathcal{K} \sim 1/\mathcal{V}$ and $\partial_5 \mathcal{K} \sim \tau_5^{1/2}/\mathcal{V} \sim 1/\mathcal{V}^{2/3}$. Since at the minimum of the scalar potential $D_i W = 0$ for the dilaton and complex structure moduli, the volume dependence of F-terms is given by

$$\begin{aligned} F^4 &\sim \frac{1}{\mathcal{V}} \left(\frac{\mathcal{V}}{\mathcal{V}} + \frac{\mathcal{V}^{2/3}}{\mathcal{V}^{2/3}} \right) \sim \frac{1}{\mathcal{V}} \\ F^5 &\sim \frac{1}{\mathcal{V}} \left(\frac{\mathcal{V}^{4/3}}{\mathcal{V}^{2/3}} + \frac{\mathcal{V}^{2/3}}{\mathcal{V}} \right) \sim \frac{1}{\mathcal{V}^{1/3}} \\ F^S &\sim \frac{1}{\mathcal{V}} \left(\frac{1}{\mathcal{V}} + \frac{1}{\mathcal{V}^2} \right) \sim \frac{1}{\mathcal{V}^2}\,. \end{aligned} \tag{5.12}$$

We see that, at large volume, $\mathcal{V}^{-1/3} \sim F^5 \gg F^4 \sim \mathcal{V}^{-1} \gg F^S \sim \mathcal{V}^{-2}$.

The F-terms corresponding to complex structure moduli vanish since $\mathcal{K}^{\bar{U}i} = 0$ for i ranging over the dilaton and Kähler moduli, even after including α' corrections, and also $D_U W = 0$ at the minimum of the scalar potential if we only turn on ISD fluxes.

5.3 D3 branes

5.3.1 Scalar masses

To calculate the masses of D3 moduli it is sufficient to work with a low energy theory only containing Kähler moduli and D3 moduli, since ISD fluxes do not give masses to D3 moduli whereas they give large masses ($\mathcal{O}(m_{3/2})$) to the dilaton, the complex structure moduli and the D7 brane moduli ([2, 4, 6]). We will also restrict to a single D3-brane rather than a stack, and assume that the D3 moduli metric is diagonal for simplicity of calculations — this is of course probably untrue for the Calabi-Yau under consideration, but since we are primarily concerned with the volume scaling of the soft parameters, the features we obtain ought to be quite generic.

Equations (5.1) and (5.2) determine the way D3 moduli enter the Kähler potential. Concentrating on a single D3 modulus ϕ, the Kähler potential after integrating out the dilaton and complex structure moduli becomes

$$\mathcal{K} = -2\log\left[(T_5+\bar{T}_5 - c|\phi|^2)^{3/2} - (T_4+\bar{T}_4 - d|\phi|^2)^{3/2} + \frac{\xi'}{2}\right] + \mathcal{K}_{cs} + 2\log(36)\,, \quad (5.13)$$

Here $\xi' = 36\xi$ and c, d parametrise our ignorance of the forms ω_α and are expected to be $\mathcal{O}(1)$. This is obtained from the original Kähler potential

$$\mathcal{K} = -2\log\left(\mathcal{V}+\frac{\xi}{2}\right) + \mathcal{K}_{cs} = -2\log\left(\tau_5^{3/2} - \tau_4^{3/2} + \frac{9\sqrt{2}\xi}{2}\right) + 2\log\left(9\sqrt{2}\right).$$

The superpotential is

$$W = \hat{W} = \frac{g_s^2}{\sqrt{4\pi}}\left(W_0 + A_4 e^{-i\frac{a_4}{g_s}T_4} + A_5 e^{-i\frac{a_5}{g_s}T_5}\right), \quad (5.14)$$

there being no supersymmetric μ-term for D3 brane scalars. We will assume for simplicity of expressions that W is real.

After expanding $\mathcal{K}$ around $\phi = 0$ we get the following expressions for $\tilde{\mathcal{K}}_i$ and $\hat{\mathcal{K}}$:

$$\begin{aligned}\tilde{\mathcal{K}}_i &= 3\frac{c(T_5+\bar{T}_5)^{1/2} - d(T_4+\bar{T}_4)^{1/2}}{(T_5+\bar{T}_5)^{3/2} - (T_4+\bar{T}_4)^{3/2} + \frac{\xi'}{2}}\,,\\ \hat{\mathcal{K}} &= -2\log\left[(T_5+\bar{T}_5)^{3/2} - (T_4+\bar{T}_4)^{3/2} + \frac{\xi'}{2}\right]. \end{aligned} \quad (5.15)$$

We now introduce the variables $X = (T_4+\bar{T}_4)^{1/2}, Y = (T_5+\bar{T}_5)^{1/2}$ to simplify various expressions appearing in the rest of this section.

It is important to note that in the no-scale approximation (obtained by setting $\xi = 0$ in $\mathcal{K}$ and $A_4 = A_5 = 0$ in W), the nonvanishing F-terms are $F^4 = -e^{\hat{\mathcal{K}}/2}X^2 W$, $F^5 = -e^{\hat{\mathcal{K}}/2}Y^2 W$ (this is derived in appendix C). We also have, after a redefinition of c and d,

$$\log(\tilde{\mathcal{K}}_i) = \log(cX + dY) - \log(\mathcal{V}) + \text{const.} \quad (5.16)$$

It is easy to check that $F^m \bar{F}^{\bar{n}} \partial_m \partial_{\bar{n}} \log(cX+dY) = -e^{\hat{\mathcal{K}}}|W|^2/2$. Also

$$F^m \bar{F}^{\bar{n}} \partial_m \partial_{\bar{n}} \log(\mathcal{V}) = -\left(\frac{1}{2}\right)\hat{\mathcal{K}}_{m\bar{n}} F^m \bar{F}^{\bar{n}} = -\left(\frac{3}{2}\right) e^{\hat{\mathcal{K}}}|W|^2 \quad (5.17)$$

in this approximation, so that

$$F^m \bar{F}^{\bar{n}} \partial_m \partial_{\bar{n}} \log\tilde{\mathcal{K}}_i = e^{\hat{\mathcal{K}}}|W|^2\,. \quad (5.18)$$

This cancels against $m_{3/2}^2 = e^{\hat{\mathcal{K}}}|W|^2$ in the expression for soft masses (5.6) giving the no-scale result $m_i^2 = 0$.

Let us estimate the size of soft scalar masses in the $\mathbb{P}^4_{[1,1,1,6,9]}$ model, without doing an explicit calculation. Consider the expression (5.6). The inclusion of α' and nonperturbative effects alters the F-terms (through $\hat{\mathcal{K}}$, $\hat{\mathcal{K}}^{\bar{T}_i T_j}$ and $\partial_i W$) and the expression for $\tilde{\mathcal{K}}_i$. After including nonperturbative contributions (but temporarily neglecting α' corrections) the F-terms are

$$\begin{aligned} F^4 &\sim \frac{1}{\mathcal{V}} + \frac{1}{\mathcal{V}} \mathcal{K}^{T_4 \bar{T}_4} (\partial_4 W) \\ F^5 &\sim \frac{1}{\mathcal{V}^{1/3}} + \frac{1}{\mathcal{V}} \mathcal{K}^{T_4 \bar{T}_5} (\partial_4 W) . \end{aligned} \tag{5.19}$$

By construction, the modulus τ_4 is small at the minimum, while τ_5 is exponentially large, so we are justified in including only the nonperturbative contribution from τ_4. Moreover, τ_4 is such that at the (AdS) minimum we have

$$-\partial_{\tau_4} W = \frac{a_4 A_4}{g_s} e^{-\frac{a_4}{g_s}\tau_4} \sim \frac{\xi^{\frac{1}{3}} |W_0|}{\langle \mathcal{V}_s \rangle} , \tag{5.20}$$

and so $(\partial_4 W) \sim 1/\mathcal{V}$. Therefore, we may use the expressions for the inverse metric $\mathcal{K}^{\bar{T}_i T_j}$ given in (5.11) to write

$$\begin{aligned} F^4 &\sim \frac{1}{\mathcal{V}} + \frac{1}{\mathcal{V}} , \\ F^5 &\sim \frac{1}{\mathcal{V}^{1/3}} + \frac{1}{\mathcal{V}^{4/3}} . \end{aligned} \tag{5.21}$$

In these expressions the second term corresponds to the modification coming from the non-perturbative addition to the superpotential. The dominant contributions to scalar masses will come from terms of the form $F^m_{np} \bar{F}^{\bar{n}} \partial_m \partial_{\bar{n}} \log(\tilde{\mathcal{K}}_i)$ where F^m_{np} is the nonperturbative contribution to the F-term. Finally we have to see how $\partial_m \partial_{\bar{n}} \log \tilde{\mathcal{K}}_i$ scales with the volume, for $m, n \in \{4, 5\}$. Using the explicit expressions (C.7) derived in the appendix we see that

$$\begin{aligned} \partial_4 \partial_4 \log(cX + dY) &\sim \mathcal{V}^{-1/3} , \\ \partial_4 \partial_5 \log(cX + dY) &\sim \frac{1}{\mathcal{V}} , \\ \partial_5 \partial_5 \log(cX + dY) &\sim \mathcal{V}^{-4/3} . \end{aligned} \tag{5.22}$$

Using the above expressions for F-terms and (5.22), we have

$$\begin{aligned} F^4_{np} \bar{F}^{\bar{4}} \partial_4 \partial_{\bar{4}} \log(cX + dY) &\sim \frac{1}{\mathcal{V}} \frac{1}{\mathcal{V}} \mathcal{V}^{-1/3} = \mathcal{V}^{-7/3} , \\ F^4_{np} \bar{F}^{\bar{5}} \partial_4 \partial_{\bar{5}} \log(cX + dY) &\sim \frac{1}{\mathcal{V}} \frac{1}{\mathcal{V}^{4/3}} \mathcal{V}^{-1} = \mathcal{V}^{-10/3} , \\ F^5_{np} \bar{F}^{\bar{5}} \partial_5 \partial_{\bar{5}} \log(cX + dY) &\sim \frac{1}{\mathcal{V}^{4/3}} \frac{1}{\mathcal{V}^{1/3}} \mathcal{V}^{-4/3} = \mathcal{V}^{-3} . \end{aligned} \tag{5.23}$$

We therefore expect the masses squared to be $\mathcal{O}(1/\mathcal{V}^{7/3})$. A similar analysis can be done for the terms of type $F^m_{np} \bar{F}^{\bar{n}} \hat{\mathcal{K}}_{m\bar{n}}$ and also including α' corrections — these turn out to be subleading compared to the contribution from nonperturbative corrections to the superpotential. The explicit expression for moduli masses can be found in appendix C.

Our conclusion is that the masses of D3 moduli are suppressed with respect to the gravitino mass, by a factor $\mathcal{V}^{-1/6}$ (the factors of g_s and W_0 also present are derived in appendix C):

$$m_i = \mathcal{O}(1)\frac{g_s^2 W_0}{\sqrt{4\pi}(\mathcal{V}_s^0)^{7/6}}\,. \tag{5.24}$$

5.3.2 Gaugino masses

We next turn our attention to gaugino masses for D3 branes. The gauge kinetic function is $f = \mu_3 l^2 S = S/(2\pi)$ and the normalised gaugino masses

$$M_{D3} = \frac{1}{2}2\pi(\mathrm{Re}\, S)^{-1}F^S = \mathcal{O}\left(\frac{1}{\mathcal{V}^2}\right). \tag{5.25}$$

F^S must be calculated using the full Kähler potential, before integrating out the dilaton and complex structure moduli. M_{D3} turns out to be proportional to g_s^2 and W_0 in the same way scalar masses are — this can be deduced from the $1/g_s$ prefactor in the inverse metric $\mathcal{K}^{\bar{S}T_i}$ (as shown in [48]) and the factor of $g_s^{3/2}$ in W, which can be observed in (4.14).

5.3.3 A-terms

We can also estimate the magnitude of the A-terms which vanish in the no-scale approximation. If we use (5.15) in the A-term expression (5.8), with constant Y_{ijk}, we obtain

$$\begin{aligned} A_{\phi\phi\phi} = e^{\hat{\mathcal{K}}/2}\Bigg\{ -\frac{3\xi}{4\mathcal{V}}W + (\partial_4 W)\Bigg[\frac{X^2}{36\mathcal{V}}(2Y^3 + X^3 - \frac{\xi'}{2}) - \frac{3Y}{\mathcal{V}}X^2Y^2 \\ + \frac{3}{2(cX+dY)}\left(\frac{c}{3}(2Y^3 + X^3 - \xi'/2) + dX^2Y\right)\Bigg]\Bigg\}. \end{aligned} \tag{5.26}$$

As $\partial_4 W \sim \mathcal{O}(1/\mathcal{V})$, $A \sim Y^2/\mathcal{V}^2 \sim \mathcal{V}^{-4/3}$. Similarly to the scalar and gaugino masses, the dependence of A on g_s and W_0 is given by by $A \propto g_s^2 W_0$.

5.3.4 μ-terms and B-terms

For D3 branes, the supersymmetric μ term vanishes, but there is an effective μ-term generated by the Giudice-Masiero mechanism [49] due to the appearance of a bilinear in the Kähler potential dependent on complex structure moduli, as follows from formulae (5.1) and (5.2). The prefactor of the bilinear $\phi^i\phi^j$ in the expansion of the Kähler potential is

$$Z_{ij} = \frac{3\mu_3 l^2}{\mathcal{V}+\xi/2}t^\alpha(\omega_\alpha)_{(j|\bar{s}}(\bar{\chi}_{\hat{a}})^{\bar{s}}_{|i)}\bar{U}^{\hat{a}}\,. \tag{5.27}$$

For simplicity we consider only one Z with $Z = (c'X + d'Y)(a_i U^i)/(\mathcal{V}+\xi/2)$. The complex structure moduli dependence of Z is in fact unimportant since the F-terms corresponding to these moduli are vanishing. The calculation of the effective μ-term will be very similar to the computation of A-terms; we do not need to differentiate by U, and can absorb $a_i U^i$ into c' and d' to write $Z = (c'X + d'Y)/(\mathcal{V}+\xi/2)$. Then

$$\begin{aligned} \partial_4 Z &\sim \left(\frac{1}{\mathcal{V}}\right), \\ \partial_5 Z &\sim \frac{1}{\mathcal{V}^{4/3}} \end{aligned} \tag{5.28}$$

Scale	Mass	GUT	Intermediate	TeV
Scalars m_i	$\frac{g_s^2}{(\mathcal{V}_s^0)^{7/6}} W_0 M_P$	3.6×10^{11} GeV	3.6×10^4GeV	3.6×10^{-17}GeV
Gauginos M_{D3}	$\frac{g_s^2}{(\mathcal{V}_s^0)^2} W_0 M_P$	3.6×10^9GeV	3.6×10^{-3}GeV	3.6×10^{-39} GeV
A-term A	$\frac{g_s^2}{(\mathcal{V}_s^0)^{4/3}} W_0 M_P$	3.2×10^{11}GeV	3.2×10^3 GeV	3.2×10^{-21}GeV
μ-term $\hat{\mu}$	$\frac{g_s^2}{(\mathcal{V}_s^0)^{4/3}} W_0 M_P$	3.2×10^{11}GeV	3.2×10^3 GeV	3.2×10^{-21}GeV
B term $\hat{\mu}B$	$\frac{g_s^2}{(\mathcal{V}_s^0)^{7/6}} W_0 M_P$	3.6×10^{11} GeV	3.6×10^4GeV	3.6×10^{-17}GeV

Table 3: Soft terms for D3 branes (AMSB contributions not included).

so that $F^m \partial_m Z$ gives rise to terms

$$\begin{aligned} e^{\mathcal{K}/2} \mathcal{K}^{44} (\partial_4 W) \partial_4 Z &\sim \frac{1}{\mathcal{V}^2}\,, \\ e^{\mathcal{K}/2} \mathcal{K}^{54} \partial_4 W \partial_5 Z &\sim \frac{1}{\mathcal{V}^{8/3}}\,. \end{aligned} \tag{5.29}$$

Assuming the complex structure moduli to be fixed at $\mathcal{O}(1)$ values, it is easy to confirm that the $\hat{\mu}$ term scales as $\mathcal{O}(1/\mathcal{V}^{4/3})$.

As Z can be treated as $(c'X + d'Y)/(\mathcal{V} + \xi/2)$ and by analogy with the calculation of the masses squared, it is easy to see that the expression for $\hat{\mu}B$ behaves like $\mathcal{O}(1/\mathcal{V}^{7/3})$.

Not including anomaly mediated contributions, the dependence of D3 brane soft terms on $\mathcal{V}, W_0$ and g_s is summarised in table 3.

5.4 D7 branes

The open string sector of D7 branes can give rise to several different types of moduli in the low energy theory. There are geometric moduli corresponding to deformations of the internal 4-cycle Σ that the D7 brane wraps. As discussed in [46], the number of such moduli is related to the $(2,0)$ cohomology of the cycle Σ. There are also Wilson line moduli a_I which will be present if the cycle Σ possesses harmonic $(1,0)$ forms. However this is not the case for most Calabi-Yaus. If present, these enter the Kähler potential through the complexified Kähler moduli, which are further redefined from (5.2) to

$$T_\alpha = \tau_\alpha + i\rho_\alpha + i\mu_3 l^2 (\omega_\alpha)_{i\bar{j}} \operatorname{Tr} \phi^i \left(\bar{\phi}^{\bar{j}} - \frac{i}{2} \bar{U}^{\hat{a}} (\bar{\chi}_{\hat{a}})^{\bar{j}}_l \phi^l \right) + \mu_7 l^2 C^{I\bar{J}} a_I \bar{a}_{\bar{J}}\,, \tag{5.30}$$

for geometry dependent coefficients $C^{I\bar{J}}$. The D7 geometric moduli are generically given large, $\mathcal{O}(m_{3/2})$ masses after turning on fluxes— this is most easily seen from the F-theory perspective, where the D7 moduli are among the complex structure moduli of the Calabi-Yau 4-fold M_8 of the F-theory compactification. The relevant Gukov-Vafa-Witten superpotential is

$$W = \int_{M_8} G_4 \wedge \Omega\,, \tag{5.31}$$

which generically induces a nontrivial potential for the D7 moduli. We expect the α' and nonperturbative effects not to affect the already large D7 scalar masses by very much.

JHEP08(2005)007

For the case of a single geometric D7 brane modulus, the Kähler potential is $-\log(S + \bar{S} - \mathcal{L}|\zeta|^2)$ giving $\tilde{\mathcal{K}} = \mathcal{L}/(S+\bar{S})$. As F^S vanishes before breaking the no-scale structure, for D7 branes $F^m \bar{F}^{\bar{n}} \partial_m \partial_{\bar{n}} \log \tilde{\mathcal{K}}$ vanishes and the flux-induced mass of ζ is $\mathcal{O}(m_{3/2})$ (which is of order $1/\mathcal{V}$). Including α' corrections, F^S is no longer zero and

$$F^S \bar{F}^{\bar{S}} \partial_S \partial_{\bar{S}} (-\log(S+\bar{S})) = \frac{1}{(S+\bar{S})^2} F^S \bar{F}^{\bar{S}} = \mathcal{O}\left(\frac{1}{\mathcal{V}^4}\right). \tag{5.32}$$

This is a manifestly tiny correction to m_ζ, and so

$$m_\zeta \approx m_{3/2} = \mathcal{O}(1) \frac{g_s^2 W_0}{\sqrt{4\pi} \mathcal{V}_s^0} M_P . \tag{5.33}$$

The masses of Wilson line moduli for D7 branes (if present) may be found by using the modified Kähler coordinates (5.30). Since these moduli appear in the Kähler potential in a similar fashion to D3 moduli, the calculation of their masses squared will be exactly parallel. We therefore expect the Wilson line moduli to obtain $\mathcal{O}(\mathcal{V}^{-7/6})$ masses due to nonperturbative effects:

$$m_{\text{Wilson}} = \mathcal{O}(1) \frac{g_s^2 W_0}{\sqrt{4\pi} (\mathcal{V}_s^0)^{7/6}} M_P . \tag{5.34}$$

For D7 branes, the gauge kinetic function f_a is given by the Kähler modulus, T_a, of the cycle the D-branes wrap:

$$f_a = \frac{T_a}{2\pi} .$$

In general $F^{T^a} \neq 0$ and the gaugino masses are nonvanishing. For the $\mathbb{P}^4_{[1,1,1,6,9]}$ example, if $a = 4$ then

$$M_4 = \frac{\pi F^4}{(\mathrm{Re}\, T_4)} \sim \mathcal{V}^{-1} , \tag{5.35}$$

and if $a = 5$ we get

$$M_5 \sim \frac{\mathcal{V}^{-1/3}}{\mathcal{V}^{2/3}} = \mathcal{V}^{-1} . \tag{5.36}$$

In either case

$$M_{D7} = \mathcal{O}(1) \frac{g_s^2 W_0}{\sqrt{4\pi} \mathcal{V}_s^0} . \tag{5.37}$$

Another crucial difference between D3 and D7 branes is that a supersymmetric μ-term can be induced for geometric D7 moduli with only ISD fluxes; in fact, it was shown in [5] that for vanishing magnetic fluxes on the D7 brane, the μ-term corresponds to the $(2,1)$ component of flux G_3. This will give an extra contribution to the mass of D7 geometric moduli, to be added to the SUSY breaking contribution proportional to the $(0,3)$ flux component. However, for simplicity, we only assume the latter to be present.

Let us now consider the A-term corresponding to a D7 scalar field ϕ with $\tilde{\mathcal{K}}_i = \tilde{\mathcal{K}}_j = \tilde{\mathcal{K}}_k = \mathcal{L}/(S+\bar{S})$. We have

$$A_{\phi\phi\phi} = F^4(\partial_4 \hat{\mathcal{K}}) + F^5(\partial_5 \hat{\mathcal{K}}) + F^S(\partial_S \hat{\mathcal{K}}) + \frac{3F^S}{S+\bar{S}} . \tag{5.38}$$

Scale	Mass	GUT	Intermediate	TeV
Scalars m_ζ	$m_{3/2}$	1.5×10^{12} GeV	1.5×10^{6}GeV	1.5×10^{-12}GeV
Gauginos M_4, M_5	$m_{3/2}$	1.5×10^{12} GeV	1.5×10^{6}GeV	1.5×10^{-12}GeV
A-term A	$m_{3/2}$	1.5×10^{12} GeV	1.5×10^{6}GeV	1.5×10^{-12}GeV
μ-term $\hat{\mu}$	$m_{3/2}$	1.5×10^{12} GeV	1.5×10^{6}GeV	1.5×10^{-12}GeV
B term $\hat{\mu}B$	$m_{3/2}$	1.5×10^{12} GeV	1.5×10^{6}GeV	1.5×10^{-12}GeV

Table 4: Soft terms for D7 branes (AMSB not included).

As $\partial_S \hat{\mathcal{K}} = -1/(S+\bar{S}) + \mathcal{O}(\mathcal{V}^{-1})$ and $F^S \sim 1/\mathcal{V}^2$, the largest contribution to the D7 A-term comes from $F^5(\partial_5 \mathcal{K})$,and so

$$A \sim F^5(\partial_5 K) \sim \frac{1}{\mathcal{V}^{1/3}} \cdot \frac{1}{\mathcal{V}^{2/3}} = \frac{1}{\mathcal{V}} \,. \tag{5.39}$$

Thus the A-term is $\mathcal{O}(m_{3/2})$.

If we try and realise the Standard Model on D7 branes, there is one potential worry. The Yang-Mills gauge coupling is determined by $g_{YM,a}^{-2} = \mathrm{Re}\ f_a$, and thus if $f_a = T^5$ then g_{YM} would be unacceptably small. However, in general, as shown in section 4.6, we have $T_b \gg 1$ but $T_{s,i}$ relatively small. Thus so long as the Standard Model is realised on branes wrapping the smaller cycles, the resulting gauge kinetic function will have phenomenologically acceptable values near 1.

5.5 D3-D7 strings

The intersections of D3 and D7 branes can give rise to massless open string states. Unfortunately their appearance in the Kähler potential of the low energy theory cannot be deduced from the dimensional reduction of Dirac-Born-Infeld and Chern-Simons actions for stacks for branes, and thus their soft terms are difficult to analyse from the four-dimensional point of view. However, in [5], the soft masses for 3-7 scalar fields due to a general flux background were computed using various symmetry arguments. It was found that no scalar or fermion masses are generated. In some particular examples where it is known how the 3-7 scalars enter the 4D Kähler potential this can be seen from the four dimensional perspective. In [50, 7] the Kähler potential was obtained for compactifications on $T^2 \times T^2 \times T^2$ with D3 and D7 branes. The dependence on 3-7 fields ϕ_{37} is

$$\mathcal{K} = \frac{|\phi_{37}|^2}{(T_1 + T_1^*)^{1/2}(T_2 + T_2^*)^{1/2}} + \cdots \,, \tag{5.40}$$

for a D7 brane wrapping the first two tori. If there is only one overall Kähler modulus $T = T_1 = T_2$ this becomes

$$\mathcal{K} = \frac{|\phi_{37}|^2}{T + \bar{T}} + \cdots \,, \tag{5.41}$$

and the same no-scale cancellation argument of section 5.3.1 for D3 matter applies for ϕ_{37}.

As for D3 matter, after including no-scale breaking effects, we expect the masses of 3-7 fields to be

$$m_{37} = \mathcal{O}(1) \frac{g_s^2 W_0}{\sqrt{4\pi} (\mathcal{V}_s^0)^{7/6}} \,. \tag{5.42}$$

Although not done explicitly, the A-terms for D3-D7 matter could be computed by setting some of the $\tilde{K}$ to equal $\mathcal{L}/(S+\bar{S})$ in formula (5.8).

5.6 D-terms and de Sitter lifting

5.6.1 D-terms and soft supersymmetry breaking

To perform a semirealistic computation of the soft supersymmetry breaking terms, we need to uplift the nonsupersymmetric AdS vacuum obtained by fixing the moduli using α' corrections and nonperturbative effects. This can be done in several ways, all of which share some essential features. One option, put forward in [3] is to use an anti-D3 brane at the bottom of a highly warped Klebanov-Strassler throat to generate the uplifting term. Although the resulting uplifting term is reminiscent of a D-term in the low energy theory, supersymmetry is broken explicitly, albeit by a small amount. An alternative means of uplifting to de Sitter was proposed in [21], where one turns on magnetic fluxes on a D7 brane wrapping a compact 4-cycle in the Calabi-Yau. The uplifting term thus generated can indeed be interpreted as a D-term in the low energy theory [51]. Again, one needs the brane to be in a highly warped region to be able to fine tune the resulting cosmological constant. Yet another mechanism for producing an uplifting term was proposed in [22]: instead of having a strongly warped region, one can look for local minima of the no-scale potential and find one with $V_0 \neq 0$ (for a contrary view, see [52]). This will happen for certain non-ISD choices of fluxes; for example, in a model with only one Kähler modulus T, setting $W = \int G_3 \wedge \Omega$ gives a source term for T

$$V_0 = \frac{1}{(S+\bar{S})(T+\bar{T})^3} \left| \int G_3^* \wedge \Omega \right|^2 . \tag{5.43}$$

V_0 will be nonzero if we turn on a non-ISD $(3,0)$ component in G_3. The large number of complex structure moduli and hence choices of fluxes generically present in Calabi-Yau compactifications should enable us to find a G_3 with sufficiently small $\int G_3^* \wedge \Omega$. Finally, one could also use the ideas of [53].

Irrespective of the uplift mechanism used, extra contributions to scalar masses will appear. For concreteness let us use an uplift appropriate to IASD fluxes

$$D = \frac{\epsilon}{\mathcal{V}^2} = \frac{\epsilon}{[(T_5+\bar{T}_5 - c|\phi|^2)^{3/2} - (T_4+\bar{T}_4 - d|\phi|^2)^{3/2}]^2} , \tag{5.44}$$

expressed in terms of the complexified Kähler moduli T_4 and T_5. This may be expanded around $\phi = 0$ as

$$D = \frac{\epsilon}{[(T_5+\bar{T}_5)^{3/2} - (T_4+\bar{T}_4)^{3/2}]^2} \left(1 + \frac{3|\phi|^2 (c(T_5+\bar{T}_5)^{3/2} - d(T_4+\bar{T}_4)^{3/2})}{(T_5+\bar{T}_5)^{3/2} - (T_4+\bar{T}_4)^{3/2}} \right) . \tag{5.45}$$

Ignoring α' corrections, the coefficient of $|\phi|^2$ in the brackets can be identified with the prefactor of kinetic term for the ϕ fields, (5.15), and so the uplift gives a contribution $\epsilon/\mathcal{V}^2$ to scalar masses squared. To uplift to Minkowski space we require $\langle D \rangle = \langle V_{\rm min} \rangle \sim \mathcal{O}(1/\mathcal{V}^3)$ which gives $\epsilon \sim 1/\mathcal{V}$. Thus the uplift contribution to scalar masses squared is $\mathcal{O}(1/\mathcal{V}^3)$, much less than the $\mathcal{O}(\mathcal{V}^{-7/3})$ obtained from nonperturbative and α' effects.

If IASD fluxes are present,[11] we no longer have $F^U = 0$. However, if the IASD fluxes are to serve as a lifting term, we can estimate the magnitude of the complex structure F-terms. We require

$$\frac{e^{\mathcal{K}_{cs}}\mathcal{K}_{cs}^{\bar{\imath}j}(D_{\bar{\imath}}W)(D_jW)}{\mathcal{V}^2} \sim \frac{1}{\mathcal{V}^3}\,,$$

and so $D_iW \sim \mathcal{V}^{-1/2}$ It then follows that $F^U \sim \mathcal{V}^{-\frac{3}{2}}$ and so the resulting effect on the masses is subleading to the F-terms associated with the Kähler moduli.

5.6.2 Uplifting to de Sitter space and numerical estimates

We wish to investigate whether the AdS minima of [14] can be lifted to de Sitter ones using a D-term and how that changes the values of the Kähler moduli at the minimum. The lifting potential is assumed to be of the form

$$V_{\text{uplift}} = \frac{\epsilon}{\mathcal{V}^{4/3}}\,. \tag{5.46}$$

This corresponds to the result ϵ/T^2 in [54], where ϵ is essentially equal to the exponentially small factor $e^{-8\pi K/(3g_sM)}$ at the bottom of the Klebanov-Strassler type throat and T is an overall radial modulus. The value of the scalar potential at the AdS minimum is $\mathcal{O}(1/\mathcal{V}^3)$ so that we want ϵ to be of order $\mathcal{O}(1/\mathcal{V}^{5/3})$. Concentrating on our 2-modulus toy model, we expect that the values of τ_4 and $\mathcal{V}$ at the minimum of the scalar potential should not change by too much after lifting to de Sitter. To see this, recall that the scalar potential before including the D-term roughly has the form

$$V = \frac{\lambda\sqrt{\tau_4}e^{-2a_4\tau_4}}{\mathcal{V}} - \frac{\mu}{\mathcal{V}^2}\tau_4 e^{-a_4\tau_4} + \frac{\nu}{\mathcal{V}^3}\,. \tag{5.47}$$

At the minimum one has

$$\mathcal{V} = \mathcal{V}_0 = \frac{\mu}{\lambda}\sqrt{\tau_4}e^{a_4\tau_4}\left(1 \pm \sqrt{1 - \frac{3\nu\lambda}{\mu^2\tau_4^{3/2}}}\right). \tag{5.48}$$

Also, in the approximation $a_4\tau_4 \gg 1$, we have

$$\tau_4 = \left(\frac{4\nu\lambda}{\mu^2}\right)^{2/3}. \tag{5.49}$$

Thus if we add an uplift term of the form $\epsilon/\mathcal{V}^{4/3}$ with $\epsilon \sim 1/\mathcal{V}_0^{5/3}$, near $\mathcal{V} = \mathcal{V}_0$ we can write the total scalar potential as in (5.47) but with ν shifted to $\nu + \epsilon{\mathcal{V}_0}^{5/3}$. To see how this works in practice, we pick the values $a_4 = 2\pi/10, A_4 = 1, g_s = 1/10, W_0 = 10, \xi = 1.31$, giving $\tau_4 = 4.17$ and $\mathcal{V}_0 = 4.5 \times 10^{10}$. We include a D-term with $\epsilon = 4.9 \times 10^{-19}$ and get a de Sitter minimum with $\tau_4 = 4.213$ and $\mathcal{V} = 5.9 \times 10^{10}$ and with value of the scalar potential 2.5×10^{-35}. Note that $\epsilon \cdot {\mathcal{V}_0}^{5/3} \approx 0.28$ so the numerics agree with our estimates from the analytic formulae. Also note that since $\mathcal{V}$ depends on τ_4 exponentially, an $\mathcal{O}(1)$ change in τ_4 can induce a change in $\mathcal{V}$ of almost an order of magnitude.

[11] We thank S. Kachru for reminding us of this possibility.

Scale	$\mathcal{V}_s$	g_sN	N if $g_s = 0.1$
GUT	4600	2.25	22
Intermediate	4.6×10^9	0.85	9
TeV	4.6×10^{27}	0.30	3

Table 5: Ranks of gaugino condensation gauge groups required to obtain various string scales.

From the size of ϵ necessary to lift the cosmological constant to zero, we can also esimate the size of the warping at the bottom of the Klebanov-Strassler throat: recall the D-term is $e^{4A_{\min}}/\mathcal{V}^2$ which has to cancel against $\mathcal{O}(1/\mathcal{V}^3)$, so that $e^{A_{\min}} \sim \mathcal{V}^{-1/4}$. In our numerical example this is roughly 2×10^{-3} - not particularly large.

The gravitino mass is 1.1×10^5GeV, while the scalar masses for D3 moduli will be of the order $\mathcal{V}^{-7/6}$, which is 1.8TeV. This number may be further lowered due to warping if the standard model branes reside in the strongly warped region - all masses at the bottom of the warped throat are supressed by $e^{A_{\min}}$.

One more important question we can analyse numerically is how difficult it is to obtain each of the scales (GUT, intermediate, TeV) in the two moduli model. The size of the volume is largely determined by the ratio a_4/g_s, as can be seen from (4.6). For gaugino condensation on D7 branes,

$$a_4 = \frac{2\pi}{N} \tag{5.50}$$

so that it is really the product g_sN which determines $\mathcal{V}$ at the minimum. Fixing g_s to be $1/10$, we can calculate the N required to obtain each of the three string scales. The results are shown in table 5.

5.7 Comparison with anomaly mediated contributions

In a general model with hidden sector SUSY breaking, scalar masses, gaugino masses and A-terms are generated through loop effects as a consequence of the super-Weyl anomaly ([55]). Assuming that soft terms are generated solely through anomaly mediation, their values are

$$\begin{aligned} M &= \frac{\beta_g}{g} m_{3/2}\,, \\ m_i^2 &= -\frac{1}{4}\left(\frac{\partial \gamma_i}{\partial g}\beta_g + \frac{\partial \gamma_i}{\partial y}\beta_y\right) m_{3/2}^2\,, \\ A_y &= -\frac{\beta_y}{y} m_{3/2}\,, \end{aligned} \tag{5.51}$$

for gaugino mass M and scalar masses m_i. Here β are the relevant β functions, γ_i are the anomalous dimensions of the chiral superfields, while g and y denote the gauge and Yukawa couplings, respectively. Alternatively one can use the compensator field formalism and write

$$M = \frac{bg^2}{8\pi^2}\frac{F^C}{C_0}\,,$$

JHEP08(2005)007

$$A_{ijk} = -\frac{\gamma_i + \gamma_j + \gamma_k}{16\pi^2}\frac{F^C}{C_0}\,,$$
$$m_i^2 = \frac{1}{32\pi^2}\frac{d\gamma_i}{d\log\mu}\left|\frac{F^C}{C_0}\right|^2 + \frac{1}{16\pi^2}\left(\gamma_m^i F^m\left(\frac{F^C}{C_0}\right)^*\right)\,, \tag{5.52}$$

where C is the compensator field and $\gamma_m^i \equiv \partial_m \gamma^i$. We expect $F^C/C_0 \sim m_{3/2}^2$ The contribution to scalar masses from AMSB is then (see [56])

$$m \sim b_0\left(\frac{g^2}{16\pi^2}\right)m_{3/2}, \tag{5.53}$$

where b_0 is the one loop beta function coefficient — this is suppressed with respect to the gravitino mass by a factor $1/(16\pi^2)$. Similarly the gaugino masses are suppresed by $1/(8\pi^2)$. As the nonperturbatively generated scalar masses $\mathcal{O}(m_{3/2}/\mathcal{V}^{7/6})$, the AMSB contribution starts to compete with the nonperturbative superpotential contribution to the masses squared at the string scale of about $m_s = 2\times 10^{10}$ GeV. The possibility of competition of the two contributions in the context of the KKLT scenario was analysed in [10].

This seems interesting, since scenarios incorporating the MSSM with purely AMSB generated masses suffer from a common problem — negative masses squared for the scalar superpartners. In our case, the $\mathcal{V}^{-7/6}$ contribution seems to have the ability to cure the problem, but it might lead to difficulties with FCNC, if the contribution is generation dependent. The nonperturbative contributions to gaugino masses are generically very small and can only start competing with AMSB induced masses at string scales of order 5×10^{14}GeV or above.

6. Differences with KKLT vacua

Before concluding let us compare our results with the more standard KKLT vacua.

1. First, as pointed out in section 3, the range of validity of the KKLT effective action is very limited because the perturbative corrections to the Kähler potential tend to dominate both at large and small volume. This is due to the original no-scale property of the Kähler potential. The KKLT potential is only valid in the regime where the flux-induced tree level superpotential is comparable to or less than its non-perturbative corrections. Otherwise, the perturbative corrections dominate and α' corrections must be included to trust any results. Including these corrections, the resulting potential includes both the large-volume minimum discussed above and, for very small values of W_0, the KKLT minimum. When $W_0 \ll 1$ the two minima coexist, and decreasing W_0 causes the two minima to approach each other and to eventually merge. This behaviour is illustrated in figures 3 and 4.

2. In the KKLT approach it is common for the potential to develop tachyonic directions after fixing the Kähler moduli. The reason was explained in [14]. The dilaton (S) and complex structure (U) moduli are fixed by a term $\frac{DW\cdot DW(S,U)}{\mathcal{V}^2}$, but the minimum

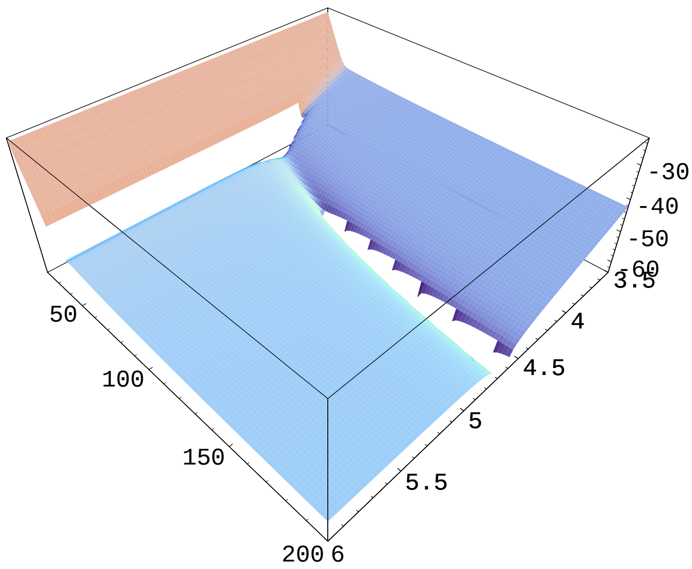

Figure 3: A plot of $\ln V$, showing the region of the scalar potential in which the large volume minimum coexists with a KKLT minimum at smaller volume. This picture is valid for very small W_0 ($\sim 10^{-10}$).

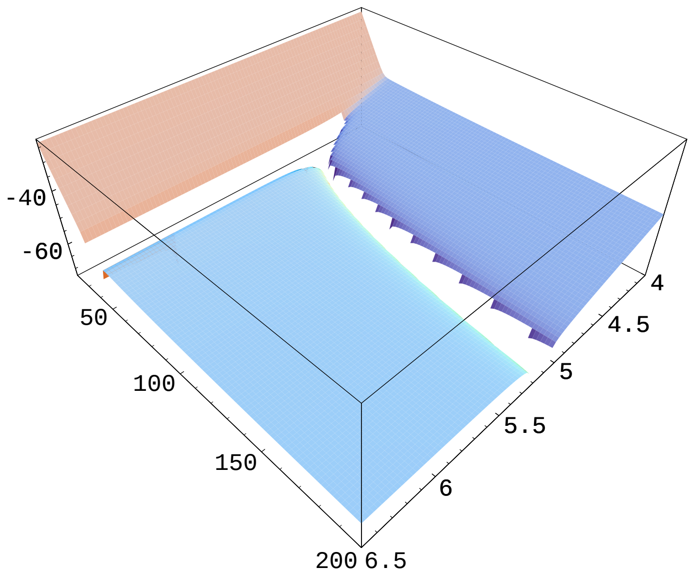

Figure 4: A plot of $\ln V$, showing how as W_0 is further decreased the large volume minimum merges with the KKLT minimum.

of the potential is at $-\frac{e^{\mathcal{K}_{cs}}|W|^2(S,U)}{\mathcal{V}^2}$. As the volume scaling of these terms is the same, the negative part of the potential may trigger a destabilization of one or more directions.

For the large-volume minimum, the minimum of the potential is at $-\frac{e^{\mathcal{K}_{cs}}|W|^2(S,U)}{\mathcal{V}^3}$. As the contribution from dilaton and complex structure F-terms is still positive and of $\mathcal{O}(\frac{1}{\mathcal{V}^2})$, movement of these moduli from $D_iW = 0$. could only increase the potential.

Therefore unlike KKLT there are no tachyonic directions in the geometric moduli. This is the main reason the models above are far simpler to analyse regarding soft supersymmetry breaking. In the KKLT scenario, it has so far not been possible to minimise the full potential, without following the two step procedure in which dilaton and complex structure moduli are fixed by the fluxes and then integrated out. This procedure fails in many cases giving rise to tachyonic directions. This was explicitly seen in [8, 9] for the simplest cases with no complex structure moduli.[12] For more complicated cases, the explicit dependence of the Kähler potential on the complex structure moduli must be written in expansions around the Gepner or conifold point and many terms are needed in order to check that the full matrix of second derivatives has only positive eigenvalues.

3. As emphasised in [14], given fixed g_s, the gravitino mass is essentially independent of the value of W_0. This is relevant for the recent interest in the scale of supersymmetry breaking in landscape scenarios. This differs from the KKLT scenario, in which the gravitino mass depends linearly on W_0 and is a statistical variable, even at fixed g_s.

4. The main difference lies in the supersymmetry breaking effects. In KKLT, the AdS minimum is supersymmetric. and the entire source of supersymmetry breaking terms is in the uplift mechanism. The lifting term dominates the soft supersymmetry breaking terms in the sense that all F-terms vanish if this term is absent. In our scenario the original minimum is already non-supersymmetric AdS, and the sources of supersymmetry breaking are principally the Kähler moduli F-terms. As discussed in section 5, these give the dominant contribution to soft terms irrespective of the uplift mechanism.

7. Conclusions and outlook

We have initiated a detailed phenomenological analysis of a large class of Calabi-Yau models with all geometric moduli fixed. The moduli spectrum and D3/D7 soft terms, both computed here, are the starting point for any phenomenology of realistic models.

One remarkable result is that the volume is fixed at an exponentially large value, which is the first time that large extra dimensions have been realised in string theory. Furthermore, we have argued that in the general case, there is one dominant large Kähler modulus whereas the others are smaller with sizes near the string scale (although large enough to neglect, e.g. higher instanton corrections). By large we mean large enough to obtain naturally, for instance, the electroweak scale from string theory (and thus solving the hierarchy problem dynamically). Indeed, it is possible to obtain even larger extra dimensions corresponding to string scales below the electroweak scale. A possible way of

[12] In this case however, a simple modification of the system can be made that stabilises the moduli [9]. Non perturbative effects from D3 branes can induce a superpotential of the form e^{-bS} which can be added to the KKLT superpotential, so $W = W_0 + Ae^{-aT} + Be^{-bS}$. This gives rise to a minimum on the full S, T plane. Adding D3 branes in principle adds new moduli corresponding to the position of the D3 branes but these are also fixed as it can be seen that the corresponding masses for D3 brane scalars are positive.

JHEP08(2005)007

getting submillimetre scales as in the six-dimensional large extra dimensions scenarios was mentioned in section 4 but this remains to be obtained in an explicit compactification. In any case electroweak or intermediate string scales are in some sense easier to obtain than larger scales, since smaller values of N are required. However, all scales from the TeV to the Planck mass are possible. The main tuning parameter ($g_s N$ for the two-moduli example) only need vary within a range of $0.3 - 3$ to cover all physically accessible string scales. For a value of $g_s \sim 0.1$ we need $N = 9$ to have an intermediate string scale and $N = 3$ to have the TeV scale (where N is the rank of the gauge group). Thus in these models there is no hierarchy problem. It is interesting that for fixed g_s arbitrarily small scales are not really accessible, since we need $N \geq 2$ to have non-trivial gauge dynamics on the hidden D7 branes in this case.

A more technical result concerns the actual calculability of soft supersymmetry breaking terms. This has proved difficult in the standard KKLT scenario due to the generic appearance of tachyons. These are all absent in our models. There are several scenarios for realistic soft breaking terms, depending on the string scale and the location of the standard model fields on either D3 or D7 branes. For D3 branes the bulk-induced soft terms are hierarchically smaller than the gravitino mass. This realises several proposals in the past (for recent discussons of this class of models see [57, 58]). For D7 branes the dominant contribution comes from the flux induced soft terms which are all of $\mathcal{O}(m_{3/2})$. The difference with previous results in the no-scale approximation is that here the volume is fixed dynamically.

It is interesting that intermediate string scales ($m_s \sim 10^{10-12}\,\mathrm{GeV}$) are naturally preferred to obtain TeV scale soft terms and stabilise the Higgs mass. The phenomenological virtues of these scales were discussed in [59, 60]. Nevertheless a GUT scale scenario may give rise to the right scale of soft breaking terms by including, for example, warping effects. Furthermore, even the TeV scenario may be realised if the standard model is inside an anti-brane or feels supersymmetry breaking directly. Here, as pointed out in section 4, the problem with the small mass of the volume modulus would have to be addressed.

This opens new avenues for phenomenological studies of realistic string models. In particular, the bottom-up approach to string model building proposed in [61] fits well with our scenario. The recent models constructed in warped Calabi-Yau compactifications naturally realise the Standard Model in our scenario. It would be interesting to explicitly combine our results with realistic models such as in [61 − 65] and to explore further the phenomenology of the different scenarios defined by the value of the string scale. Furthermore, the structure of soft breaking terms we have found has to be complemented, in the large string scale scenarios, with a low-energy analysis using the renormalisation group equations to extrapolate the low-energy implications of our soft terms. A recent analysis in this direction, using only the flux-induced supersymmetry breaking, can be found in [66].

There may be interesting cosmological implications. In all cases there exist extremely flat directions, such as the one corresponding to the volume axion. This is also protected by a Peccei-Quinn symmetry making it stable under quantum corrections. Ironically, in most cases this field is too light to be a candidate for a quintessence field. We may ask the question backwards and look for the value of the string scale such that this field has

JHEP08(2005)007

a mass of order 10^{-33} eV. While there is some dependence on the associated gauge group rank, this is typically close to the Planck scale ($m_s \sim 10^{17-18}$ GeV.).

A further cosmological application could be to use the volume modulus as an inflaton. Note that the potential is rather flat in the large volume limit. In particular, in the regime where our minimum coexists with the KKLT minimum, a saddle point develops separating both minima, which may eventually give rise to inflation. In particular, recent models of inflation in the KKLT scenario either coming from brane/antibrane systems [54, 67 − 73] or the racetrack inflation [74, 75], could be reconsidered in view of our results.

We must also address the overshooting problem [76] as well as the cosmological moduli problem [43, 44, 77]. Note that in the intermediate scale scenario, the volume modulus T_5 is on the border of being ruled out by this problem.

There are also other open questions. The lack of control of the value of the D7 moduli is probably the less understood part of our construction. This is in general not a serious drawback. We know that these moduli have compact support, and therefore must have a minimum. Recently it has been found using F-theory techniques that D7 moduli tend not to be stabilised at the orientifold limit [78]. It would then be interesting to work in the F-theory setting with full control over the D7 moduli.

A further study of loop corrections, both from string theory and field theory is desirable to pin down the magnitude of any effects not included here. As described in section 3, these are principally loop effects in the open string sector. D3 brane loop corrections to the Kähler potential are currently being computed in [30], and these may compete with the α'^3 corrections in the very large volume limit (see also [79]). In the regime of small coupling and large volume our solution will still be dominant but in other regimes loop corrections may be important. This may still be desirable depending on the sign of the correction. If positive, it would provide a much better lifting mechanism than any considered so far. If negative, it may drive the potential to vanish from below at infinity with α' corrections dominating at smaller (but still large) volumes, still implying the existence of a large-volume minimum. In any case we expect that our main results will be mostly maintained but it would certainly be very interesting to have control of all loop corrections to the Kähler potential.

Finally, loop corrections from the effective four-dimensional theory may modify the values of the masses we have found. This depends on the magnitude of the couplings. A preliminary analysis shows that these corrections are suppressed by the volume and therefore do not tend to destabilise the masses found above.[13] Finally, it would be desirable to have a better control of the effects of warping. Although significant warping is not necessary to obtain a hierarchy in the above models, it is very interesting that flux compactifications can realise both the Randall-Sundrum scenario [80] with exponentially large geometric hierarchies in the complex structure sector of the theory, whereas the Kähler sector realises the large extra dimensions scenario. Exploring both these effects in explicit string theoretical scenarios will certainly provide very interesting phenomenology.

[13]We thank C.P. Burgess for this estimate.

Acknowledgments

We are grateful to B. Allanach, V. Balasubramanian, D. Baumann, P. Berglund, R. Blumenhagen, C.P. Burgess, D. Cremades, G. Curio, B.de Carlos, F. Denef, B. Florea, A. Font, M.B. Green, T. House, S. Kachru, L. McAllister, E. Palti, A. Sinha, S. Stieberger and A. Uranga for interesting discussions. We particularly thank P. Cámara and L.E. Ibáñez for early collaboration on soft breaking terms. FQ thanks J.J. Blanco-Pillado for a conversation in 2003 regarding the relative importance of perturbative and non-perturbative effects in supergravity lagrangians. JC is grateful to EPSRC for a research studentship. FQ is partially funded by PPARC and a Royal Society Wolfson award. KS would like to thank Trinity College, Cambridge for financial support.

A. Dimensional reduction

The bosonic type-IIB supergravity action in string frame is [81]

$$S_{IIB} = \frac{1}{(2\pi)^7 \alpha'^4} \int d^{10}x \sqrt{-g} \left\{ e^{-2\phi} [\mathcal{R} + 4(\nabla\phi)^2] - \frac{F_1^2}{2} - \frac{1}{2 \cdot 3!} G_3 \cdot \bar{G}_3 - \frac{\tilde{F}_5^2}{4 \cdot 5!} \right\}. \quad \text{(A.1)}$$

It is convenient to work in Einstein frame. We redefine

$$g_{MN} = e^{(\phi - \phi_0)/2} \tilde{g}_{MN} \,, \quad \text{(A.2)}$$

where $\phi_0 = \langle \phi \rangle$. The factor of $e^{\frac{\phi_0}{2}}$ ensures that g and $\tilde{g}$ are identical in the physical vacuum. The action is then

$$\frac{2\pi e^{-2\phi_0}}{l_s^8} \int d^{10}x \sqrt{-\tilde{g}} \left\{ \tilde{\mathcal{R}} - \frac{\partial_M S \partial^M \bar{S}}{2(\text{Re } S)^2} - \frac{e^{\phi_0} G_3 \cdot \bar{G}_3}{12\,\text{Re } S} - \frac{e^{2\phi_0} \tilde{F}_5^2}{4 \cdot 5!} \right\}, \quad \text{(A.3)}$$

where $l_s = 2\pi\sqrt{\alpha'}$ and $S = e^{-\phi} + iC_0$. We neglect warping effects; as discussed in section 3.3, these are subleading at large volume. In the orientifold limit and in the absence of warping, $\tilde{F}_5 = 0$ and $\partial_M S = 0$. The dimensional reduction of (A.3) then gives

$$S = \frac{2\pi}{g_s^2 l_s^8} \left(\int d^4x \sqrt{-\tilde{g}_4} \tilde{\mathcal{R}}_4 \mathcal{V} - \overbrace{\int d^4x \sqrt{-\tilde{g}_4} \left(\int d^6x \sqrt{\tilde{g}_6} \frac{e^{\phi_0} G_3 \cdot \bar{G}_3}{12\,\text{Re } S} \right)}^{V_{\text{flux}}} \right), \quad \text{(A.4)}$$

where $g_s = e^{\phi_0}$ and $\mathcal{V} = \int d^6x \sqrt{\tilde{g}_6}$. In 4d Einstein frame, the Einstein-Hilbert action is

$$S_{EH} = \frac{1}{16\pi G} \int d^4x \sqrt{-g_E} \mathcal{R}_E \equiv \frac{M_P^2}{2} \int d^4x \sqrt{-g_E} \mathcal{R}_E \,. \quad \text{(A.5)}$$

$\tilde{g}_4$ and g_E are related by $\tilde{g}_4 = g_E \frac{\mathcal{V}_s^0}{\mathcal{V}_s}$, where $\mathcal{V} \equiv \mathcal{V}_s l_s^6$ and $\mathcal{V}_s^0 = \langle \mathcal{V}_s \rangle$. This gives

$$M_P^2 = \frac{4\pi \mathcal{V}_s^0}{g_s^2 l_s^2} \qquad \text{and} \qquad m_s = \frac{g_s}{\sqrt{4\pi \mathcal{V}_s^0}} M_P \,. \quad \text{(A.6)}$$

The Kähler potential is determined to be

$$\frac{\mathcal{K}}{M_P^2} = -2\ln(\mathcal{V}_s) - \ln(S+\bar{S}) - \ln\left(-i\int \Omega \wedge \bar{\Omega}\right). \tag{A.7}$$

The superpotential can be found from V_{flux}. If

$$W = \frac{1}{l_s^2}\int G_3 \wedge \Omega\,, \tag{A.8}$$

then

$$V_{\text{flux}} = \frac{4\pi(\mathcal{V}_s^0)^2}{g_s l_s^4}\int d^4x\sqrt{-g_E}e^{\mathcal{K}}\left[G^{a\bar{b}}D_aWD_{\bar{b}}W - 3W\bar{W}\right], \tag{A.9}$$

with a, b running over all moduli. Thus if

$$\hat{W} = \frac{g_s^{\frac{3}{2}}M_P^3}{\sqrt{4\pi}l_s^2}\int G_3 \wedge \Omega\,, \tag{A.10}$$

the scalar potential takes the standard $\mathcal{N}=1$ form

$$V = \int d^4x\sqrt{-g_E}e^{\mathcal{K}/M_P^2}\left[G^{a\bar{b}}D_a\hat{W}D_{\bar{b}}\bar{\hat{W}} - \frac{3}{M_P^2}\hat{W}\bar{\hat{W}}\right]. \tag{A.11}$$

The Kähler potential will receive perturbative corrections and the superpotential non-perturbative corrections. The α' corrections to the Kähler potential modify the 4-dimensional kinetic terms and arise from the higher-derivative terms in the ten-dimensional IIB action. The dimensional reduction of these gives the perturbative correction to the Kähler potential. We then obtain

$$\frac{\mathcal{K}}{M_P^2} = -2\ln(\mathcal{V}_s + \frac{\xi g_s^{3/2}}{2e^{3\phi/2}}) - \ln(S+\bar{S}) - \ln\left(-i\int \Omega \wedge \bar{\Omega}\right). \tag{A.12}$$

where $\xi = -\frac{\chi(M)\zeta(3)}{2(2\pi)^3}$. The superpotential can receive non-perturbative corrections causing it to depend on the Kähler moduli. These can arise from D3-brane instantons or gaugino condensation. The generic form of the superpotential is then

$$\hat{W} = \frac{g_s^{3/2}M_P^3}{\sqrt{4\pi}}\left(W_0 + \sum A_i e^{-a_iT_i}\right). \tag{A.13}$$

B. Canonical normalisation

We here discuss the canonical normalisation of the Kähler moduli for the 2-modulus example. The Kähler potential is given by

$$\mathcal{K} = \mathcal{K}_{cs} - 2\ln\left(\left(T_5+\bar{T}_5)\right)^{\frac{3}{2}} - \left(T_4+\bar{T}_4\right)^{\frac{3}{2}}\right) + \text{constant}\,. \tag{B.1}$$

Here $iT_4 = b_4 + i\tau_4$ and $iT_5 = b_5 + i\tau_5$. All terms depending on dilaton and complex structure moduli have been absorbed into $\mathcal{K}_{cs}$. We also recall that as $\tau_5 \gg 1$, $1/\tau_5$ serves as a good expansion parameter.

It may be verified that

$$
\begin{aligned}
\partial_{T_5}\partial_{\bar{T}_5}K &= \frac{3\tau_5^{-\frac{1}{2}}}{4(\tau_5^{\frac{3}{2}}-\tau_4^{\frac{3}{2}})} + \frac{9\tau_4^{\frac{3}{2}}\tau_5^{-\frac{1}{2}}}{8(\tau_5^{\frac{3}{2}}-\tau_4^{\frac{3}{2}})^2}\,, \\
\partial_{T_4}\partial_{\bar{T}_5}K = \partial_{T_5}\partial_{\bar{T}_4}K &= \frac{-9\tau_4^{\frac{1}{2}}\tau_5^{\frac{1}{2}}}{8(\tau_5^{\frac{3}{2}}-\tau_4^{\frac{3}{2}})}\,, \\
\partial_{T_4}\partial_{\bar{T}_4}K &= \frac{3\tau_4^{-\frac{1}{2}}}{4(\tau_5^{\frac{3}{2}}-\tau_4^{\frac{3}{2}})} + \frac{9\tau_4}{8(\tau_5^{\frac{3}{2}}-\tau_4^{\frac{3}{2}})^2}\,.
\end{aligned}
$$

These results are summarised by

$$
G = \begin{pmatrix} G_{4\bar{4}} & G_{4\bar{5}} \\ G_{5\bar{4}} & G_{5\bar{5}} \end{pmatrix} = \begin{pmatrix} \frac{3\tau_4^{-\frac{1}{2}}}{8\tau_5^{\frac{3}{2}}} + \mathcal{O}(\frac{1}{\tau_5^3}) & \frac{-9\tau_4^{\frac{1}{2}}}{8\tau_5^{\frac{5}{2}}} + \mathcal{O}(\frac{1}{\tau_5^4}) \\ \frac{-9\tau_4^{\frac{1}{2}}}{8\tau_5^{\frac{5}{2}}} + \mathcal{O}(\frac{1}{\tau_5^4}) & \frac{3}{4\tau_5^2} + \mathcal{O}(\frac{1}{\tau_5^{\frac{7}{2}}}) \end{pmatrix}. \tag{B.2}
$$

We denote the values of the fields $(b_4, b_5, \tau_4, \tau_5)$ at the minimum by $(b_4^0, b_5^0, \tau_4^0, \tau_5^0)$ and now define

$$
\tau_5^{'} = \sqrt{\frac{3}{2}}\frac{\tau_5}{\tau_5^0}\,, \qquad \tau_4^{'} = \sqrt{\frac{3}{4}}\frac{\tau_4}{(\tau_5^0)^{\frac{3}{4}}(\tau_4^0)^{\frac{1}{4}}}\,, \qquad b_5^{'} = \sqrt{\frac{3}{2}}\frac{b_5}{\tau_5^0}\,, \qquad b_4^{'} = \sqrt{\frac{3}{4}}\frac{b_4}{(\tau_5^0)^{\frac{3}{4}}(\tau_4^0)^{\frac{1}{4}}}\,,
$$

and

$$
\tau_4^{'0} = \sqrt{\frac{3}{4}}\frac{\tau_4^0}{(\tau_5^0)^{\frac{3}{4}}(\tau_4^0)^{\frac{1}{4}}}\,, \qquad \tau_5^{'0} = \sqrt{\frac{3}{2}}\,.
$$

Then after some manipulation, we obtain

$$
\begin{aligned}
\sum_{i,j} G_{i\bar{j}}\partial_\mu T^i \partial^\mu \bar{T}^j = \frac{1}{2}\Bigg[& \left(\frac{\tau_5^{'0}}{\tau_5^{'}}\right)^{\frac{3}{2}}\left(\frac{\tau_4^{'0}}{\tau_4^{'}}\right)^{\frac{1}{2}}\partial_\mu b_4^{'}\partial^\mu b_4^{'} + \left(\frac{\tau_5^{'0}}{\tau_5^{'}}\right)^2 \partial_\mu b_5^{'}\partial^\mu b_5^{'} - \\
& - 4\sqrt{3}\left(\frac{\tau_4^{'}}{\tau_4^{'0}}\right)^{\frac{1}{2}}\left(\frac{\tau_5^{'0}}{\tau_5^{'}}\right)^{\frac{5}{2}}\tau_4^{'0}\partial_\mu b_4^{'}\partial^\mu b_5^{'} + \\
& + \left(\frac{\tau_5^{'0}}{\tau_5^{'}}\right)^{\frac{3}{2}}\left(\frac{\tau_4^{'0}}{\tau_4^{'}}\right)^{\frac{1}{2}}\partial_\mu \tau_4^{'}\partial^\mu \tau_4^{'} + \left(\frac{\tau_5^{'0}}{\tau_5^{'}}\right)^2 \partial_\mu \tau_5^{'}\partial^\mu \tau_5^{'} - \\
& - 4\sqrt{3}\left(\frac{\tau_4^{'}}{\tau_4^{'0}}\right)^{\frac{1}{2}}\left(\frac{\tau_5^{'0}}{\tau_5^{'}}\right)^{\frac{5}{2}}\tau_4^{'0}\partial_\tau \tau_4^{'}\partial^\tau \tau_5^{'}\Bigg].
\end{aligned}
$$

The moduli are now canonically normalised except for the crossterm, which is suppressed by $(\tau_5^0)^{\frac{3}{4}}$ and is thus very small. All such crossterms could be eliminated by field redefinitions order by order in $1/\tau_5$; however, negelecting the subleading corrections it is sufficient to use $\tau_5^{'}$ and $\tau_4^{'}$ as canonically normalised fields.

C. Soft terms

Let us introduce the notation $\mathcal{V}' = (T_5+\bar{T}_5)^{3/2} - (T_4+\bar{T}_4)^{3/2}$ (which differs by a factor of 36 from the Calabi-Yau volume $\mathcal{V}$). We write

$$\begin{aligned} \hat{\mathcal{K}}_{T_5} &= -3\frac{(T_5+\bar{T}_5)^{1/2}}{\mathcal{V}'+\xi'/2}\,, \\ \hat{\mathcal{K}}_{T_4} &= 3\frac{(T_4+\bar{T}_4)^{1/2}}{\mathcal{V}'+\xi'/2}\,. \end{aligned} \tag{C.1}$$

We denote $(T_4+\bar{T}_4)^{1/2} = X$[14] and $(T_5+\bar{T}_5)^{1/2} = Y$. Then $\mathcal{V}' = Y^3 - X^3$ and we can calculate the metric $\mathcal{K}_{T_i\bar{T}_j}$:

$$\mathcal{K}_{T_i\bar{T}_j} = \begin{pmatrix} \frac{3}{2Xx} + \frac{9X^2}{2x^2} & -\frac{9XY}{2x^2} \\ -\frac{9XY}{2x^2} & \frac{-3}{2Yx} + \frac{9Y^2}{2x^2} \end{pmatrix}, \tag{C.2}$$

and the inverse metric $\mathcal{K}^{\bar{T}_iT_j}$

$$\begin{pmatrix} \frac{-2X(\mathcal{V}'+\xi/2)(2Y^3+X^3-\xi'/2)}{3(\xi'/2-2\mathcal{V}')} & -2X^2Y^2\frac{(\mathcal{V}'+\xi'/2)}{(\xi'/2-2\mathcal{V}')} \\ -2X^2Y^2\frac{(\mathcal{V}'+\xi'/2)}{(\xi'/2-2\mathcal{V}')} & \frac{-2Y(\mathcal{V}'+\xi'/2)(2X^3+Y^3+\xi'/2)}{3(\xi'/2-2\mathcal{V}')} \end{pmatrix}. \tag{C.3}$$

In these expressions $x = Y^3 - X^3 + \xi'/2 = \mathcal{V}' + \xi'/2$. We can now calculate the F-terms as given by formula (5.5). We assume that $\partial_5 W \ll \partial_4 W$ (where $\partial_i \equiv \partial_{T_i}$) and only include the nonperturbative contribution corresponding to T_4. The result is

$$\begin{aligned} F^4 &= e^{\hat{\mathcal{K}}/2}\frac{2\mathcal{V}'+\xi'}{2\mathcal{V}'-\xi'/2}\left(-X^2W + \frac{X}{3}(2Y^3+X^3-\frac{\xi}{2})(\partial_4 W)\right) \\ F^5 &= e^{\hat{\mathcal{K}}/2}\frac{2\mathcal{V}'+\xi}{2\mathcal{V}'-\xi'/2}\left(-Y^2W + X^2Y^2(\partial_4 W)\right)\right). \end{aligned} \tag{C.4}$$

After a redefinition of c and d, the prefactor of the kinetic term for the brane modulus ϕ, $\tilde{\mathcal{K}}_i$,can be rewritten as

$$\tilde{\mathcal{K}}_i = \frac{cX+dY}{\mathcal{V}'+\xi'/2}\,. \tag{C.5}$$

To calculate scalar masses we need $\partial_m\partial_{\bar{n}}\log\tilde{\mathcal{K}}_i = \partial_m\partial_{\bar{n}}(\log(cX+dY) - \log(\mathcal{V}'+\frac{\xi'}{2}))$. As $\hat{\mathcal{K}} = -2\log(\mathcal{V}'+\xi'/2) + \text{const.}$, $-\partial_m\partial_{\bar{n}}\log(\mathcal{V}'+\frac{\xi'}{2}) = (1/2)\hat{\mathcal{K}}_{m\bar{n}}$ and so

$$\partial_m\partial_{\bar{n}}\log\tilde{\mathcal{K}}_i = \partial_m\partial_{\bar{n}}(\log(cX+dY)) + \frac{1}{2}\hat{\mathcal{K}}_{m\bar{n}}\,. \tag{C.6}$$

The necessary derivatives of $\log(cX+dY)$ can be calculated using $\partial_4 X = 1/(2X), \partial_5 Y = 1/(2Y)$, and likewise with respect to $\bar{4}$ and $\bar{5}$. The results are

$$\begin{aligned} \partial_4\partial_{\bar{4}}\log(cX+dY) &= -\frac{c(2cX+dY)}{4X^3(cX+dY)^2}\,, \\ \partial_4\partial_{\bar{5}}\log(cX+dY) &= -\frac{cd}{4XY(cX+dY)^2}\,, \\ \partial_5\partial_{\bar{5}}\log(cX+dY) &= -\frac{d(2dY+cX)}{4Y^3(cX+dY)^2}\,. \end{aligned} \tag{C.7}$$

[14]We do not simply use τ_4 as this would not be valid when T_4 includes D3-brane moduli.

We now wish to find the soft masses, expressing the result as a small deviation from the no-scale result, which is zero. To do this we make an expansion assuming that $(\partial_4 W)$ is small (we know it is $\mathcal{O}(1/\mathcal{V})$ at the minimum of the scalar potential) and $\xi/\mathcal{V}$ is small. For example, we can note that $F^m \bar{F}^{\bar{n}}$ always has a prefactor of $(2\mathcal{V}' + \xi')^2/(2\mathcal{V}' - \xi'/2)^2$ which may be expanded as

$$1 + \frac{3\xi'}{2\mathcal{V}'} + \mathcal{O}\left(\frac{1}{\mathcal{V}^2}\right). \tag{C.8}$$

From the above analysis we expect that $F^m \bar{F}^{\bar{n}} \partial_m \partial_{\bar{n}} \log(cX + dY)$ can be written as $e^{\hat{\mathcal{K}}}((-1/2)|W|^2 + \mathcal{O}(1/\mathcal{V}^\alpha))$ for some $\alpha > 0$. Indeed, an explicit computation gives

$$\begin{aligned}
&F^m \bar{F}^{\bar{n}} \partial_m \partial_{\bar{n}} \log(cX + dY) = \\
&\quad = e^{\hat{\mathcal{K}}} \left(1 + \frac{3\xi'}{2\mathcal{V}'}\right) \left(-\frac{1}{2}|W|^2 + \frac{1}{3(cX+dY)} \left(-\frac{\xi c}{2} + 3dX^2Y + cX^3 + 2cY^3\right) W(\partial_4 W)\right).
\end{aligned}$$

Similarly we have

$$F^m \bar{F}^{\bar{n}} \hat{\mathcal{K}}_{m\bar{n}} = e^{\hat{\mathcal{K}}} \frac{3\mathcal{V}'(2\mathcal{V}' - \frac{\xi'}{2})}{2(\mathcal{V}' + \frac{\xi'}{2})^2} \left(|W|^2 - \frac{W(\partial_4 W)}{3\mathcal{V}'} \left(2X^2 \frac{\xi'}{2} + 2X^2 \mathcal{V}'\right)\right). \tag{C.9}$$

Expanding $\mathcal{V}'(2\mathcal{V}' - \xi'/2)/(2(\mathcal{V}' + \xi'/2)^2)$ as $1 - 3\xi'/(4\mathcal{V}')$ we get

$$\begin{aligned}
F^m \bar{F}^{\bar{n}} \partial_m \partial_{\bar{n}} \log \tilde{\mathcal{K}}_i &= e^{\hat{\mathcal{K}}} \left(1 + \frac{3\xi'}{2\mathcal{V}'}\right) \times \\
&\times \left[-|W|^2 + \frac{1}{3(cX+dY)} \left(-\frac{\xi' c}{2} + 3dX^2Y + cX^3 + 2cY^3\right) W(\partial_4 W) - \right. \\
&\left. - W(\partial_4 W) X^2 (1 + \frac{\xi'}{2\mathcal{V}'})\right].
\end{aligned} \tag{C.10}$$

Further simplifying and neglecting $\mathcal{O}(1/\mathcal{V}^2)$ terms we obtain

$$e^{\hat{\mathcal{K}}} \left(-|W|^2 + \frac{3\xi'}{2\mathcal{V}'}|W|^2 + \frac{W}{3(cX+dY)} \left(\frac{-\xi' c}{2} + 3dX^2Y + cX^3 + 2cY^3\right)(\partial_4 W) - W(\partial_4 W) X^2\right).$$

The $-e^{\hat{\mathcal{K}}}|W|^2$ cancels with $m_{3/2}^2$ so the mass squared of the brane modulus can be seen to have the form

$$\frac{3\xi'}{2\mathcal{V}'} + f(X,Y)(\partial_4 W), \tag{C.11}$$

which is what we expect naively — before the no-scale structure is broken, the mass is simply zero, and there are two sources of no-scale breaking: α' corrections corresponding to the first term in (C.11) and nonperturbative corrections corresponding to the second term. To estimate the size of the soft terms, note that all the constants involved in (C.11) are expected to be $\mathcal{O}(1)$ (including c, d, ξ') and so the first term is $\mathcal{O}(1/\mathcal{V})$. As the volume is $\mathcal{V} \sim Y^3$, we can estimate

$$f(u,v) \sim \frac{Y^3}{Y} = Y^2 \sim \mathcal{V}^{2/3}. \tag{C.12}$$

We also note that, at the minimum, $\partial_4 W \sim \frac{W_0}{\mathcal{V}}$. Then the volume scaling of the second term is $\mathcal{V}^{2/3}/\mathcal{V} = \mathcal{V}^{-1/3}$, and as $e^{\hat{\mathcal{K}}} \sim 1/\mathcal{V}^2$ the moduli masses squared scale as $\mathcal{O}(1/\mathcal{V}^{7/3})$. Finally, by considering the form of the superpotential (5.14), we see that $W(\partial_4 W) \sim W_0^2 g_s^4$. Putting the factors together, we get

$$m_i^2 = \mathcal{O}(1)\frac{g_s^4 W_0^2}{4\pi(\mathcal{V}_s^0)^{7/3}} M_P^2\,. \tag{C.13}$$

References

[1] A. Strominger, *Superstrings with torsion*, *Nucl. Phys.* **B 274** (1986) 253;
B. de Wit, D.J. Smit and N.D. Hari Dass, *Residual supersymmetry of compactified $D = 10$ supergravity*, *Nucl. Phys.* **B 283** (1987) 165;
K. Becker and M. Becker, *M-theory on eight-manifolds*, *Nucl. Phys.* **B 477** (1996) 155 [hep-th/9605053];
S. Sethi, C. Vafa and E. Witten, *Constraints on low-dimensional string compactifications*, *Nucl. Phys.* **B 480** (1996) 213 [hep-th/9606122];
T.R. Taylor and C. Vafa, *RR flux on Calabi-Yau and partial supersymmetry breaking*, *Phys. Lett.* **B 474** (2000) 130 [hep-th/9912152];
B.R. Greene, K. Schalm and G. Shiu, *Warped compactifications in M and F theory*, *Nucl. Phys.* **B 584** (2000) 480 [hep-th/0004103];
C.S. Chan, P.L. Paul and H.L. Verlinde, *A note on warped string compactification*, *Nucl. Phys.* **B 581** (2000) 156 [hep-th/0003236];
M. Grana and J. Polchinski, *Supersymmetric three-form flux perturbations on AdS_5*, *Phys. Rev.* **D 63** (2001) 026001 [hep-th/0009211];
S.S. Gubser, *Supersymmetry and F-theory realization of the deformed conifold with three-form flux*, hep-th/0010010;
A.R. Frey and J. Polchinski, *$N = 3$ warped compactifications* *Phys. Rev.* **D 65** (2002) 126009;
P.K. Tripathy and S.P. Trivedi, *Compactification with flux on K3 and tori*, *JHEP* **03** (2003) 028 [hep-th/0301139];
K. Dasgupta, G. Rajesh and S. Sethi, *M-theory, orientifolds and G-flux*, *JHEP* **08** (1999) 023 [hep-th/9908088];
Two recent reviews are: V. Balasubramanian, *Accelerating universes and string theory*, *Class. Quant. Grav.* **21** (2004) S1337–1358 [hep-th/0404075];
E. Silverstein, *TASI/PITP/ISS lectures on moduli and microphysics*, hep-th/0405068.

[2] S.B. Giddings, S. Kachru and J. Polchinski, *Hierarchies from fluxes in string compactifications*, *Phys. Rev.* **D 66** (2002) 106006 [hep-th/0105097].

[3] S. Kachru, R. Kallosh, A. Linde and S.P. Trivedi, *de Sitter vacua in string theory*, *Phys. Rev.* **D 68** (2003) 046005 [hep-th/0301240].

[4] P.G. Camara, L.E. Ibáñez and A.M. Uranga, *Flux-induced SUSY-breaking soft terms*, *Nucl. Phys.* **B 689** (2004) 195 [hep-th/0311241].

[5] P.G. Camara, L.E. Ibáñez and A.M. Uranga, *Flux-induced SUSY-breaking soft terms on D7-D3 brane systems*, *Nucl. Phys.* **B 708** (2005) 268 [hep-th/0408036].

[6] M. Grana, T.W. Grimm, H. Jockers and J. Louis, *Soft supersymmetry breaking in Calabi-Yau orientifolds with D-branes and fluxes*, *Nucl. Phys.* **B 690** (2004) 21 [hep-th/0312232].

JHEP08(2005)007

[7] D. Lüst, S. Reffert and S. Stieberger, *Flux-induced soft supersymmetry breaking in chiral type-IIB orientifolds with D3/D7-branes*, *Nucl. Phys.* **B 706** (2005) 3 [hep-th/0406092].

[8] K. Choi, A. Falkowski, H.P. Nilles, M. Olechowski and S. Pokorski, *Stability of flux compactifications and the pattern of supersymmetry breaking*, *JHEP* **11** (2004) 076 [hep-th/0411066].

[9] P. Cámara, L.E. Ibáñez, F. Quevedo and K. Suruliz, unpublished.

[10] K. Choi, A. Falkowski, H.P. Nilles and M. Olechowski, *Soft supersymmetry breaking in KKLT flux compactification*, *Nucl. Phys.* **B 718** (2005) 113 [hep-th/0503216];
K. Choi, K.S. Jeong and K.-i. Okumura, *Phenomenology of mixed modulus-anomaly mediation in fluxed string compactifications and brane models*, hep-ph/0504037.

[11] M. Endo, M. Yamaguchi and K. Yoshioka, *A bottom-up approach to moduli dynamics in heavy gravitino scenario: superpotential, soft terms and sparticle mass spectrum*, *Phys. Rev.* **D 72** (2005) 015004 [hep-ph/0504036].

[12] F. Denef and M.R. Douglas, *Distributions of flux vacua*, *JHEP* **05** (2004) 072 [hep-th/0404116].

[13] F. Denef, M.R. Douglas, B. Florea, A. Grassi and S. Kachru, *Fixing all moduli in a simple F-theory compactification*, hep-th/0503124.

[14] V. Balasubramanian, P. Berglund, J.P. Conlon and F. Quevedo, *Systematics of moduli stabilisation in Calabi-Yau flux compactifications*, *JHEP* **03** (2005) 007 [hep-th/0502058].

[15] V. Balasubramanian and P. Berglund, *Stringy corrections to Kähler potentials, SUSY breaking and the cosmological constant problem*, *JHEP* **11** (2004) 085 [hep-th/0408054].

[16] N. Arkani-Hamed, S. Dimopoulos and G.R. Dvali, *The hierarchy problem and new dimensions at a millimeter*, *Phys. Lett.* **B 429** (1998) 263 [hep-ph/9803315];
Phenomenology, astrophysics and cosmology of theories with sub-millimeter dimensions and TeV scale quantum gravity, *Phys. Rev.* **D 59** (1999) 086004 [hep-ph/9807344];
I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G.R. Dvali, *New dimensions at a millimeter to a fermi and superstrings at a TeV*, *Phys. Lett.* **B 436** (1998) 257 [hep-ph/9804398].

[17] F. Denef, M.R. Douglas and B. Florea, *Building a better racetrack*, *JHEP* **06** (2004) 034 [hep-th/0404257].

[18] S. Gukov, C. Vafa and E. Witten, *CFT's from Calabi-Yau four-folds*, *Nucl. Phys.* **B 584** (2000) 69 [hep-th/9906070], erratum *ibid.* **B608** (2001) 477.

[19] E. Witten, *Non-perturbative superpotentials in string theory*, *Nucl. Phys.* **B 474** (1996) 343 [hep-th/9604030].

[20] J.P. Derendinger, L.E. Ibáñez and H.P. Nilles, *On the low-energy* $D = 4$, $N = 1$ *supergravity theory extracted from the* $D = 10$, $N = 1$ *superstring*, *Phys. Lett.* **B 155** (1985) 65;
M. Dine, R. Rohm, N. Seiberg and E. Witten, *Gluino condensation in superstring models*, *Phys. Lett.* **B 156** (1985) 55;
C.P. Burgess, J.P. Derendinger, F. Quevedo and M. Quiros, *On gaugino condensation with field-dependent gauge couplings*, *Ann. Phys. (NY)* **250** (1996) 193 [hep-th/9505171].

[21] C.P. Burgess, R. Kallosh and F. Quevedo, *de Sitter string vacua from supersymmetric D-terms*, *JHEP* **10** (2003) 056 [hep-th/0309187].

JHEP08(2005)007

[22] A. Saltman and E. Silverstein, *The scaling of the no-scale potential and de Sitter model building*, *JHEP* **11** (2004) 066 [hep-th/0402135].

[23] S. Ashok and M.R. Douglas, *Counting flux vacua*, *JHEP* **01** (2004) 060 [hep-th/0307049].

[24] F. Denef and M.R. Douglas, *Distributions of nonsupersymmetric flux vacua*, *JHEP* **03** (2005) 061 [hep-th/0411183].

[25] R. Blumenhagen, F. Gmeiner, G. Honecker, D. Lüst and T. Weigand, *The statistics of supersymmetric D-brane models*, *Nucl. Phys.* **B 713** (2005) 83 [hep-th/0411173].

[26] B.S. Acharya, F. Denef and R. Valandro, *Statistics of M-theory vacua*, hep-th/0502060.

[27] M.B. Green and S. Sethi, *Supersymmetry constraints on type-IIB supergravity*, *Phys. Rev.* **D 59** (1999) 046006 [hep-th/9808061].

[28] K. Becker, M. Becker, M. Haack and J. Louis, *Supersymmetry breaking and α'-corrections to flux induced potentials*, *JHEP* **06** (2002) 060 [hep-th/0204254].

[29] M.B. Green and C. Stahn, *D3-branes on the coulomb branch and instantons*, *JHEP* **09** (2003) 052 [hep-th/0308061].

[30] M. Berg, M. Haack and B. Körs, *Loop corrections to volume moduli and inflation in string theory*, *Phys. Rev.* **D 71** (2005) 026005 [hep-th/0404087].

[31] P. Candelas, A. Font, S. Katz and D.R. Morrison, *Mirror symmetry for two parameter models, 2*, *Nucl. Phys.* **B 429** (1994) 626 [hep-th/9403187].

[32] A. Giryavets, S. Kachru, P.K. Tripathy and S.P. Trivedi, *Flux compactifications on Calabi-Yau threefolds*, *JHEP* **04** (2004) 003 [hep-th/0312104].

[33] A. Giryavets, S. Kachru and P.K. Tripathy, *On the taxonomy of flux vacua*, *JHEP* **08** (2004) 002 [hep-th/0404243].

[34] J.P. Conlon and F. Quevedo, *On the explicit construction and statistics of Calabi-Yau flux vacua*, *JHEP* **10** (2004) 039 [hep-th/0409215].

[35] L. Görlich, S. Kachru, P.K. Tripathy and S.P. Trivedi, *Gaugino condensation and nonperturbative superpotentials in flux compactifications*, hep-th/0407130.

[36] P.K. Tripathy and S.P. Trivedi, *D3 brane action and fermion zero modes in presence of background flux*, hep-th/0503072.

[37] R. Kallosh and D. Sorokin, *Dirac action on M5 and M2 branes with bulk fluxes*, *JHEP* **05** (2005) 005 [hep-th/0501081].

[38] N. Saulina, *Topological constraints on stabilized flux vacua*, hep-th/0503125.

[39] R. Kallosh, A.-K. Kashani-Poor and A. Tomasiello, *Counting fermionic zero modes on M5 with fluxes*, hep-th/0503138.

[40] P. Berglund and P. Mayr, *Non-perturbative superpotentials in F-theory and string duality*, hep-th/0504058.

[41] T. Banks, M. Dine and E. Gorbatov, *Is there a string theory landscape?*, *JHEP* **08** (2004) 058 [hep-th/0309170].

JHEP08(2005)007

[42] A. Albrecht, C.P. Burgess, F. Ravndal and C. Skordis, *Exponentially large extra dimensions*, *Phys. Rev.* **D 65** (2002) 123506 [hep-th/0105261];
Y. Aghababaie, C.P. Burgess, S.L. Parameswaran and F. Quevedo, *Towards a naturally small cosmological constant from branes in 6D supergravity*, *Nucl. Phys.* **B 680** (2004) 389 [hep-th/0304256];
Y. Aghababaie et al., *Warped brane worlds in six dimensional supergravity*, *JHEP* **09** (2003) 037 [hep-th/0308064];
C.P. Burgess, J. Matias and F. Quevedo, *Msled: a minimal supersymmetric large extra dimensions scenario*, *Nucl. Phys.* **B 706** (2005) 71 [hep-ph/0404135];
C.P. Burgess, *Towards a natural theory of dark energy: supersymmetric large extra dimensions*, *AIP Conf. Proc.* **743** (2005) 417–449 [hep-th/0411140].

[43] T. Banks, D.B. Kaplan and A.E. Nelson, *Cosmological implications of dynamical supersymmetry breaking*, *Phys. Rev.* **D 49** (1994) 779 [hep-ph/9308292].

[44] B. de Carlos, J.A. Casas, F. Quevedo and E. Roulet, *Model independent properties and cosmological implications of the dilaton and moduli sectors of 4-D strings*, *Phys. Lett.* **B 318** (1993) 447 [hep-ph/9308325].

[45] P. Candelas and X. de la Ossa, *Moduli space of Calabi-Yau manifolds*, *Nucl. Phys.* **B 355** (1991) 455.

[46] H. Jockers and J. Louis, *The effective action of D7-branes in $N = 1$ Calabi-Yau orientifolds*, *Nucl. Phys.* **B 705** (2005) 167 [hep-th/0409098].

[47] A. Brignole, L.E. Ibáñez and C. Muñoz, *Soft supersymmetry-breaking terms from supergravity and superstring models*, hep-ph/9707209.

[48] K. Bobkov, *Volume stabilization via α' corrections in type-IIB theory with fluxes*, *JHEP* **05** (2005) 010 [hep-th/0412239].

[49] G.F. Giudice and A. Masiero, *A natural solution to the μ problem in supergravity theories*, *Phys. Lett.* **B 206** (1988) 480.

[50] L.E. Ibáñez, C. Muñoz and S. Rigolin, *Aspects of type-I string phenomenology*, *Nucl. Phys.* **B 553** (1999) 43 [hep-ph/9812397].

[51] H. Jockers and J. Louis, *D-terms and F-terms from D7-brane fluxes*, *Nucl. Phys.* **B 718** (2005) 203 [hep-th/0502059].

[52] S.P. de Alwis, *On potentials from fluxes*, *Phys. Rev.* **D 68** (2003) 126001 [hep-th/0307084];
S.P. de Alwis, *Brane worlds in 5D and warped compactifications in IIB*, *Phys. Lett.* **B 603** (2004) 230 [hep-th/0407126].

[53] R. Brustein and S.P. de Alwis, *Moduli potentials in string compactifications with fluxes: mapping the discretuum*, *Phys. Rev.* **D 69** (2004) 126006 [hep-th/0402088].

[54] S. Kachru et al., *Towards inflation in string theory*, *JCAP* **10** (2003) 013 [hep-th/0308055].

[55] L. Randall and R. Sundrum, *Out of this world supersymmetry breaking*, *Nucl. Phys.* **B 557** (1999) 79 [hep-th/9810155];
G.F. Giudice, M.A. Luty, H. Murayama and R. Rattazzi, *Gaugino mass without singlets*, *JHEP* **12** (1998) 027 [hep-ph/9810442].

[56] O. DeWolfe and S.B. Giddings, *Scales and hierarchies in warped compactifications and brane worlds*, *Phys. Rev.* **D 67** (2003) 066008 [hep-th/0208123].

[57] R. Kallosh and A. Linde, *Landscape, the scale of SUSY breaking and inflation*, *JHEP* **12** (2004) 004 [hep-th/0411011].

[58] R. Allahverdi, S. Hannestad, A. Jokinen, A. Mazumdar and S. Pascoli, *Supermassive gravitinos, dark matter, leptogenesis and flat direction baryogenesis*, hep-ph/0504102.

[59] K. Benakli, *Phenomenology of low quantum gravity scale models*, *Phys. Rev.* **D 60** (1999) 104002 [hep-ph/9809582].

[60] C.P. Burgess, L.E. Ibáñez and F. Quevedo, *Strings at the intermediate scale or is the fermi scale dual to the planck scale?*, *Phys. Lett.* **B 447** (1999) 257 [hep-ph/9810535].

[61] G. Aldazabal, L.E. Ibáñez, F. Quevedo and A.M. Uranga, *D-branes at singularities: a bottom-up approach to the string embedding of the standard model*, *JHEP* **08** (2000) 002 [hep-th/0005067];
G. Aldazabal, L.E. Ibáñez and F. Quevedo, *Standard-like models with broken supersymmetry from type-I string vacua*, *JHEP* **01** (2000) 031 [hep-th/9909172]; *A D-brane alternative to the MSSM*, *JHEP* **02** (2000) 015 [hep-ph/0001083].

[62] J.F.G. Cascales, M.P. Garcia del Moral, F. Quevedo and A.M. Uranga, *Realistic D-brane models on warped throats: fluxes, hierarchies and moduli stabilization*, *JHEP* **02** (2004) 031 [hep-th/0312051].

[63] J.F.G. Cascales, F. Saad and A.M. Uranga, *Holographic dual of the standard model on the throat*, hep-th/0503079.

[64] F. Marchesano and G. Shiu, *MSSM vacua from flux compactifications*, *Phys. Rev.* **D 71** (2005) 011701 [hep-th/0408059].

[65] F. Marchesano and G. Shiu, *Building MSSM flux vacua*, *JHEP* **11** (2004) 041 [hep-th/0409132].

[66] B.C. Allanach, A. Brignole and L.E. Ibáñez, *Phenomenology of a fluxed MSSM*, *JHEP* **05** (2005) 030 [hep-ph/0502151].

[67] C.P. Burgess et al., *The inflationary brane-antibrane universe*, *JHEP* **07** (2001) 047 [hep-th/0105204];
G.R. Dvali, Q. Shafi and S. Solganik, *D-brane inflation*, hep-th/0105203.

[68] G.R. Dvali and S.H.H. Tye, *Brane inflation*, *Phys. Lett.* **B 450** (1999) 72 [hep-ph/9812483].

[69] J.P. Hsu, R. Kallosh and S. Prokushkin, *On brane inflation with volume stabilization*, *JCAP* **12** (2003) 009 [hep-th/0311077];
J.P. Hsu and R. Kallosh, *Volume stabilization and the origin of the inflaton shift symmetry in string theory*, *JHEP* **04** (2004) 042 [hep-th/0402047].

[70] H. Firouzjahi and S.H.H. Tye, *Closer towards inflation in string theory*, *Phys. Lett.* **B 584** (2004) 147 [hep-th/0312020]; *Brane inflation and cosmic string tension in superstring theory*, *JCAP* **03** (2005) 009 [hep-th/0501099].

[71] C.P. Burgess, J.M. Cline, H. Stoica and F. Quevedo, *Inflation in realistic D-brane models*, *JHEP* **09** (2004) 033 [hep-th/0403119].

[72] N. Iizuka and S.P. Trivedi, *An inflationary model in string theory*, *Phys. Rev.* **D 70** (2004) 043519 [hep-th/0403203].

[73] X.-g. Chen, *Inflation from warped space*, hep-th/0501184.

JHEP08(2005)007

[74] J.J. Blanco-Pillado et al., *Racetrack inflation*, *JHEP* **11** (2004) 063 [hep-th/0406230].

[75] Z. Lalak, G.G. Ross and S. Sarkar, *Racetrack inflation and assisted moduli stabilisation*, hep-th/0503178.

[76] R. Brustein and P.J. Steinhardt, *Challenges for superstring cosmology*, *Phys. Lett.* **B 302** (1993) 196 [hep-th/9212049];
R. Brustein, S.P. de Alwis and P. Martens, *Cosmological stabilization of moduli with steep potentials*, *Phys. Rev.* **D 70** (2004) 126012 [hep-th/0408160].

[77] G.D. Coughlan, W. Fischler, E.W. Kolb, S. Raby and G.G. Ross, *Cosmological problems for the polonyi potential*, *Phys. Lett.* **B 131** (1983) 59.

[78] D. Lüst, P. Mayr, S. Reffert and S. Stieberger, *F-theory flux, destabilization of orientifolds and soft terms on D7-branes*, hep-th/0501139.

[79] I. Antoniadis, C. Bachas and E. Dudas, *Gauge couplings in four-dimensional type-I string orbifolds*, *Nucl. Phys.* **B 560** (1999) 93 [hep-th/9906039];
I. Antoniadis, C. Bachas, C. Fabre, H. Partouche and T.R. Taylor, *Aspects of type-I — type-II — heterotic triality in four dimensions*, *Nucl. Phys.* **B 489** (1997) 160 [hep-th/9608012].

[80] L. Randall and R. Sundrum, *A large mass hierarchy from a small extra dimension*, *Phys. Rev. Lett.* **83** (1999) 3370 [hep-ph/9905221]; *An alternative to compactification*, *Phys. Rev. Lett.* **83** (1999) 4690 [hep-th/9906064].

[81] J. Polchinski, *String theory*, Cambridge University Press, Cambridge 1998.